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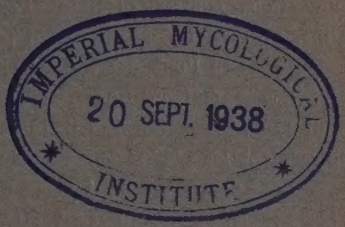

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MATHEMATICS

SOME NEW ESTIMATIONS OF THF ANALYTICAL THEORY OF NUMBERS

By I. M. VINOGRADOW, Member of the Academy

I give in the present paper some new estimations concerning the analytical theory of numbers. The complete proof will be published in an other magazine.

We introduce the following notations:

The relation

$$A \ll B$$

denotes that

$$|A| \ll cB,$$

where c is a positive constant; n denotes a constant integer > 0; $\nu = \frac{1}{n}$; $\alpha_n, \ldots, \alpha_1$ are reals; $f(x) = \alpha_n x^n + \ldots + \alpha_1 x$.

The following results may be formulated:

1. Let $n > 2$; N is an integer > 1; m is an integer > 0;

$$\alpha_n = \frac{a}{q} + \frac{\theta}{q^2}; \quad (a, q) = 1; \quad q > 0; \quad |\theta| \leqslant 1;$$

$$S = \sum_{p \leqslant N} e^{2\pi i m f(p)},$$

p runs over primes.

Then we have:

$$S \ll N \left(\frac{m^5}{N} + \frac{m^5}{q} + \frac{q^{2.5}}{N^{2.5n}} \right)^{\beta} (\log N)^{1.5 \log\log N}; \quad \beta = \frac{1}{49\, n^6 (\log n)^2},$$

and if the condition

$$\min \left(q, \frac{N^n}{q} \right) \ll e^{\sqrt{\log N}}$$

is fulfilled we have also:

$$S \ll N \left(\frac{m^2}{q} + \frac{mq}{N^n} \right)^{\gamma} (\log N)^{1.5}; \quad \gamma = \frac{1}{39.2\, n^6 (\log n)^2}.$$

This result is the improvement of my previous result in the same direction.

2. Let $n \geqslant 14$; m is an integer > 0; P is an integer > 1;

$$\alpha_n = \frac{a}{q} + \lambda q^{-2}; \quad (a, q) = 1; \quad \lambda \ll \Lambda; \quad q_1 \ll q \ll q_1;$$

$$S = \sum_{x=1}^{P} e^{2\pi i m f(x)}.$$

In my book «A New Method in the Analytical Theory of Numbers» [2] for the case *

$$P^{n-1} \leqslant q_1 \leqslant P^{n-\tau};\ \Lambda m \ll P^{\tau v^s};\ 1 \ll \tau \leqslant 1$$

the following estimation has been given:

$$S \ll P^{1-\rho};\ \rho > \frac{2\tau}{n^3\left(1+2.5\frac{\log\log n^2}{\log n^2}\right)\log\frac{n^2}{\tau}}.$$

Now I have the possibility to complete this estimation by two new ones more precise for the small values of τ:

Under the conditions $P^{n-1} \leqslant q_1 \leqslant P^{n-\tau};\ 0 \leqslant \tau \leqslant 1;\ m\Lambda \ll P$, we have

$$S \ll P^{1-\rho}(\log q)^{\frac{\rho}{\tau}};\ \rho = \frac{\tau}{8.24\, n^3 \log n},$$

and also

$$S \ll P^{1-\rho_1};\ \rho_1 = \frac{\tau}{16.48\, n^3 \log n}.$$

In my above mentioned book τ must exceed a certain positive arbitrary small constant. Now τ may tend to zero when P increases.

3. Let $n > 2$; m is an integer > 0; P is an integer > 1; s denotes one of the numbers $2, \ldots, n-1$;

$$\alpha_s = \frac{a_s}{q_s} + \frac{\theta}{q_s^2};\ (a_s, q_s) = 1;\ 0 < q_s \leqslant P^s;\ |\theta| \leqslant 1; \qquad S = \sum_{x=1}^{P} e^{2\pi i m f(x)}.$$

Then we have

$$S \ll P\left(\frac{m}{q_s} + \frac{m}{P} + \frac{q_s}{P^s}\right)^{\lambda};\ \lambda = \frac{1}{33.7\, n^6 (\log n)^2}.$$

4. Let q be an odd prime; m is a divisor of $q-1$; $1 < m \leqslant q-1$; s is one of the numbers $0, \ldots, m-1$; k is an integer and is not divisible by q; N is a positive integer; p runs over primes.

Then the number T of numbers

$$p + k$$

under the condition:

$$p \leqslant N;\ \operatorname{ind}(p+k) \equiv s \pmod m,$$

is expressed by the formula

$$T = \frac{\pi(N)}{m} + O\left(N^{1+\varepsilon}\sqrt{\frac{1}{q} + \frac{q}{N^{\frac{1}{3}}}}\right),$$

ε being arbitrary small positive constant.

This formula is the generalization of my previous results for $n = 2$

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Moskow.

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[1] Travaux de l'Institut Mathématique de Tbilissi, III (1937). [2] Travaux de l'Institut Mathématique Stekloff, X (1937).

* I use the opportunity to correct one imperfection of my above mentioned book. In theorem 1 of the chapter VII the lower bound of r, for which the asymptotical formula of Hardy-Littlewood giving the number of representations of an positive integer N in the form:

$$N = x_1^n + \ldots + x_r^n;\ n \geqslant 14,$$

takes place, must be replaced by the following:

$$r > n^3(\log n + 2.2 \log\log n).$$

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MATHEMATIK

ÜBER EINIGE DIE KONVEXEN KÖRPER CHARAKTERISIERENDE EIGENSCHAFTEN

Von J. LIEBERMANN
(Vorgelegt von I. M. Vinogradow, Mitglied der Akademie, d. 5.III.1938)

Wir sagen, dass eine Menge $U=\{u\}$ in einen Punkt zusammenziehbar ist, wenn eine auf der Menge $U=\{u\}$ bestimmte Funktion $f(u,t)$ $(0\leqslant t\leqslant 1)$ existiert derart, dass

1) $f(u,t)\in U$, $f(u,0)=u$;

2) $\lim\limits_{u'\to u} f(u',t)=f(u,t)$;

3) für alle $u\in U$ und $\varepsilon>0$ sich ein $\delta>0$ angeben lässt derart, dass aus der Ungleichung $(t'-t)<\delta$

$$\rho[f(u,t);\ f(u,t')]<\varepsilon \text{ folgt;}$$

4) $f(u,1)=w\in U$ für alle $u\in U$.

Es sei eine geschlossene und begrenzte Menge $P\subset R_n$ gegeben. R_n bedeutet den n-dimensionalen euklidischen Raum. Als eine Seitenfläche $(n-1)$-ter Ordnung (oder erste Seitenfläche) bezeichnen wir den Durchschnitt der Menge P mit der $(n-1)$-dimensionalen Stützebene.

Bestimmen wir jetzt, was wir als eine Seitenfläche $(n-k)$-ter Ordnung bezeichnen wollen.

Die Seitenfläche $(n-k)$-ter Ordnung ist die erste Seitenfläche der Seitenfläche $(n-k+1)$-ter Ordnung, als eine Untermenge des $(n-k+1)$-dimensionalen Unterraumes, in welchem sie gelegen ist, betrachtet.

Den Unterraum der allerkleinsten Dimensionalzahl, den die Seitenfläche Γ der angegebenen Ordnung enthält, nennen wir den «Träger» dieser Seite, und bezeichnen selbige mit dem Symbol $N^k(\Gamma)$, wo k die Dimensionalzahl des Trägers ist. Die Seitenfläche besitzt die folgenden offensichtlichen Eigenschaften:

1. $N^k(\Gamma)\cdot P\neq 0$ und $N^k(\Gamma)$ ist immer eine Ebene der Seitenfläche.

2. $N^0(\Gamma)$ besteht aus einem einzigen Punkte.

3. Für einen jeden Träger $N^k(\Gamma)$ und für jedes $\varepsilon>0$ existiert ein solcher Unterraum $L_{n-1}(\varepsilon)$, dass

$$\sup_{x\in L_{n-1}(\varepsilon)\cdot P}\rho[x,N^k(\Gamma)\cdot P]<\varepsilon,$$

und $N^k(\Gamma)\subset L_{n-1}(\varepsilon)$ ist.

4. Eine konvexe Hülle einer Seitenfläche ist eine Seitenfläche der konvexen Hülle von P.

Theorem. *Wenn alle $(n-1)$-dimensionalen Schnitte der Menge P in einen Punkt zusammenziehbar sind, so ist P konvex.*

Ohne die Allgemeinheit unserer Ausführungen einzuschränken, dürfen wir annehmen, dass R_n ein linearer Träger der Menge P ist. Es ist leicht einzusehen, dass alle nulldimensionalen Seitenflächen der Menge konvex sind. Nehmen wir an, dass wir die Konvexität der Seitenfläche von Ordnung k $(n-1>k\geqslant 0)$ bewiesen haben.

Nun wollen wir dasselbe für alle $(k+1)$-dimensionalen Seitenflächen Γ_{k+1} beweisen. Laut 1 können wir annehmen, dass die Ebene der Seiten-

fläche Γ_{k+1} mit $N^{k+1}(\Gamma_{k+1})$ zusammenfällt; 4 zeigt uns ausserdem, auf Grund unserer Induktionsannahme, dass die äussere Grenze der konvexen Hülle der Seitenfläche Γ_{k+1}, $AR(\Gamma_{k+1})$, als eine Untermenge $N^{k+1}(\Gamma_{k+1})$ betrachtet, unserer Menge P angehört. Beweisen wir jetzt, dass alle inneren Punkte der konvexe Hülle $V(\Gamma_{k+1})$ der Menge P angehören. In der Tat, wenn ein Punkt $x \in V(\Gamma_{k+1})$ existiert, wo $x \bar{\in} P$, so existiert auch ein $\rho > 0$ derart, dass die n-dimensionale Kugel $\mathfrak{Ш}_\rho^n(x)$ von Radius ρ und mit dem Zentrum x keine Punkte der Menge P enthält.

Es sei sup $\rho = 2\delta > 0$ und der Durchmesser $P - d(P) = D$. Nehmen wir im $N^{k+1}(\Gamma_{k+1})$ die Sphären $S_\delta(x)$, $S_{D+\delta}(x)$ und $S_{\frac{D+2\delta}{2}}(x)$ von Radien δ, $D+\delta$ und $\frac{D+2\delta}{2}$.

Die Sphären $S_\delta(x)$ und $S_{D+\delta}(x)$ bestimmen in $N^{k+1}(\Gamma_{k+1})$ eine sphärische Schicht $K(\delta, D+\delta; x)$, wobei $S_\delta(x)$, $S_{D+\delta}(x)$, $S_{\frac{D+2\delta}{2}}(x)$ und $AR(\Gamma_{k+1}) \subset$ $\subset K(\delta, D+\delta; x)$ und nicht in einen Punkt in dieser sphärischen Schicht zusammenziehbar sind.

Nehmen wir irgendeinen Punkt $y \in S_{\frac{D+2\delta}{2}}(x)$ und legen durch diesen und den Punkt x die $(n-k)$ -dimensionale Ebene $L_{n-k}(y, x)$, die zu $N^{k+1}(\Gamma_{k+1})$ normal ist. Nehmen wir ferner in dieser Ebene $L_{n-k}(y, x)$ eine Kugel $\mathfrak{Ш}_{\frac{D}{2}}^{n-k}(y)$ und betrachten

$$T(x) = \sum_{y \in S_{\frac{D+2\delta}{2}}(x)} E_z\left[z \in \mathfrak{Ш}_{\frac{D}{2}}^{n-k}(y)\right],$$

$T(x)$ ist so konstruiert, dass $\Gamma_{k+1} \subset T(x)$, $K(\delta, D+\delta; x) \subset T(x)$ und $AR(\Gamma_{k+1})$ sich nicht in einen Punkt zusammenziehen lassen (*).

Laut 3 gibt es für $\varepsilon = \frac{\delta}{2}$ ein $L_{n-1}(\varepsilon)$ derart, dass sup $\rho(x, \Gamma_{k+1}) < \frac{\delta}{2}$ $x \in L_{n-1}(\varepsilon) \cdot P$ und $L_{n-1}(\varepsilon) \supset N^{k+1}(\Gamma_{k+1})$. Demzufolge ist $L_{n-1}(\varepsilon) P \subset T(x)$ (**). Da aber $L_{n-1}(\varepsilon) P$ in einem Punkt zusammenziehbar ist, so lässt sich auch $AR(\Gamma_{k+1})$ in einen Punkt zusammenziehen. Dann aber liesse sich auf Grund von (**) $AR(\Gamma_{k+1})$ innerhalb von $T(x)$ in einen Punkt zusammenziehen, dieses widerspricht jedoch (*).

Infolgedessen ist der Punkt $x \in \Gamma_{k+1}$ und daher fällt Γ_{k+1} mit seiner konvexen Hülle zusammen. Daraus folgt $AR(P) \subset P$, dann aber, da alle Schnitte der Menge P in einen Punkt zusammenziehbar sind, ist $V(P) = P$, was zu beweisen war.

Ganz ähnlich kann man auch folgenden Satz beweisen. Es sei ein jedes A_i in einen Punkt zusammenziehbar und $A_{i+1} \subset A_i$ $(i = 1, 2, \ldots)$. Wenn die Menge $U = \prod_{i=1}^{\infty} A_i$ ist, so werden wir sie als eine schwach einfachzusammenhängende Menge bezeichnen. Ein Anteil der Menge P in einer geschlossenen ε-dicken Schicht zwischen irgendeiner Stützebene und der ihr parallelen Ebene (die auf derselben Seite von L_{n-1} wie P liegt) bezeichnen wir als eine «ε-Schicht».

Theorem. *Es sei $P \subset R_n$ geschlossen und schwach zusammenhängend.* Wenn dann ein $\delta > 0$ existiert derart, dass bei $\delta > \varepsilon > 0$ alle «ε-Schichten» schwach einfachzusammenhängend sind, so ist P ein konvexer Körper.

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MATHÉMATIQUES

SUR UN THÉORÈME DE O. SCHMIDT

Par D. KOLIANKOWSKY

(Présenté par O. J. Schmidt, de l'Académie, le 1. II. 1938)

1. Dans la présente Note, nous nous proposons de généraliser un des résultats obtenus par O. Schmidt dans son travail «Les groupes dont tous les sous-groupes sont spéciaux» (1).

On sait qu'un groupe est appelé spécial lorsqu' il représente le produit direct de ses sous-groupes de Sylow. Tous les sous-groupes d'un groupe spécial sont également des groupes spéciaux. Dans le travail mentionné ci-dessus, O. Schmidt a indiqué tous les groupes non spéciaux jouissant de la même propriété (c'est-à-dire que tous leurs sous-groupes sont spéciaux). Il a démontré que tous ces groupes sont résolubles et leur ordre peut toujours être présenté sous la forme d'un produit de deux puissances de nombres premiers. Ces résultats, obtenus par O. Schmidt, contiennent, comme cas particulier, les résultats déjà obtenus par G. A. Miller et H. C. Moreno, qui ont étudié les groupes dont tous les sous-groupes sont commutatifs (2).

Dans le travail présent nous considérons les groupes qui ne possèdent qu'une classe de sous-groupes non spéciaux. Nous allons démontrer que, de même que les groupes étudiés par O. Schmidt, ces groupes sont résolubles.

2. Démontrons le lemme suivant:

Soit B *un certain groupe et* $\mathfrak{P}$ *l'intersection maximum de deux sous-groupes de Sylow quelconques du groupe* B.

Dans ce cas, le normalisateur du sous-groupe $\mathfrak{P}$ *par rapport au groupe* B *ne peut pas être un groupe spécial.*

(Dans cette démonstration nous supposons que l'intersection est un sous-groupe proprement dit du sous-groupe de Sylow, c'est-à-dire qu'elle n'est égale ni à l'unité, ni au sous-groupe de Sylow lui-même. Nous disons que l'intersection de deux sous-groupes de Sylow est maximum, si elle n'est pas contenue dans une autre intersection plus grande des sous-groupes de Sylow du groupe donné.)

Passons à la démonstration. Soient Γ et Γ' deux sous-groupes de Sylow d'ordre p^α (p — nombre premier) du groupe B. Supposons que le sous-groupe $\mathfrak{P}$ d'ordre p^β est l'intersection de ces deux sous-groupes de Sylow. Admettons que l'intersection est maximum. Dans ces conditions, ainsi qu'il a été démontré par Burnside, le groupe B doit contenir un élément d'un ordre non divisible par p, permutable avec le sous-groupe $\mathfrak{P}$ et non permutable avec ceux des sous-groupes de Sylow, qui contiennent (3) $\mathfrak{P}$.

Supposons que ce soit l'élément Q d'ordre q (non divisible par p). Soit $\mathfrak{P}_1$ le normalisateur du sous-groupe $\mathfrak{P}$ par rapport à Γ (évidemment $\mathfrak{P}_1 \supset \mathfrak{P}$), et soit $\mathfrak{N}$ le normalisateur du sous-groupe $\mathfrak{P}$ par rapport à B. Il est évident que $\mathfrak{N}$ contient Q et $\mathfrak{P}_1$. Si $\mathfrak{N}$ est un groupe spécial, nous avons l'égalité: $Q^{-1}\mathfrak{P}_1 Q = \mathfrak{P}_1$, c'est-à-dire, $\mathfrak{P}_1$ fait partie du sous-groupe de Sylow Γ et du sous-groupe de Sylow $Q^{-1}\Gamma Q$ qui ne peut coïncider avec Γ.

Mais, dans ce cas, contrairement à l'hypothèse, $\mathfrak{P}$ ne peut pas représenter l'intersection maximum de deux sous-groupes de Sylow du groupe B. Par conséquent, $\mathfrak{N}$ ne peut être un groupe spécial et notre lemme est démontré.

3. Nous utiliserons, dans le présent article, le théorème suivant, démontré par W. K. Turkin dans son travail «Ein neues Kriterium der Einfachheit einer endlichen Gruppe» (4):

Soit B *un groupe d'ordre* n *et* Γ *un sous-groupe d'ordre* g, *c'est-à-dire* $n = gm$ (g *et* m *sont premiers entre eux*). *Soit* P *le commutant du sous-groupe* Γ. *Si* Γ *contient un élément* A, *non contenu dans le commutant* P, *tel que chaque égalité de la forme*:

$$T^{-1}A^{\lambda}T = G$$

(T *étant un élément de* B, G — *un élément de* Γ) *entraîne une égalité de la forme*:

$$X^{-1}A^{\lambda}X = G$$

(X *étant un élément de* Γ), *alors* B *possède un diviseur normal dont l'ordre est un multiple de* m.

Soit B un groupe quelconque ne possédant qu'une seule classe de sous-groupes non spéciaux. Nous devons démontrer que le groupe B est résoluble. A cet effet, il suffit de montrer que le groupe B ne peut pas être simple. En effet, nous pouvons considérer notre proposition comme démontrée pour tous les groupes d'ordre inférieur à celui du groupe B. Dans ce cas tout diviseur normal et tout groupe complémentaire du groupe donné sont résolubles, et, par conséquent, le groupe B lui-même est résoluble.

Supposons que le groupe B, ne possédant qu'une seule classe de sous-groupes non spéciaux, est un groupe simple.

Démontrons que cela nous amenera à une contradiction. Soit Γ un sous-groupe de Sylow d'ordre p^{λ} du groupe B. D'après le théorème de W. K. Turkin précédemment mentionné, il doit exister dans le sous-groupe Γ un élément P vérifiant l'égalité:

$$T^{-1}P^{\lambda}T = G$$

où T est un élément du groupe B non contenu dans Γ, et G un élément du sous-groupe Γ. D'autre part, aucun des éléments X, contenus dans Γ, ne peut vérifier l'égalité:

$$X^{-1}P^{\lambda}X = G$$

(si un tel élément P n'existait pas dans Γ, d'après le théorème mentionné le groupe B posséderait un diviseur normal).

L'égalité $T^{-1}P^{\lambda}T = G$ entraîne l'égalité: $P^{\lambda} = TGT^{-1}$, c'est-à-dire $P^{\lambda} \subset T\Gamma T^{-1}$. Examinons les deux cas possibles:

1) $T\Gamma T^{-1} \neq \Gamma$; dans ce cas, l'élément P^{λ} fait partie d'une certaine intersection maximum de deux sous-groupes de Sylow d'ordre p^{λ} du groupe B.

D'après le lemme démontré précédemment, le normalisateur de cette intersection ne peut être un groupe spécial. Par conséquent, dans le

groupe B, il doit exister un sous-groupe non spécial d'ordre $p^{\varphi}m$, où m n'est pas divisible par p. Ce sous-groupe est le normalisateur de l'intersection maximum de deux sous-groupes de Sylow, intersection, qui contient l'élément P.

2) $T\Gamma T^{-1}=\Gamma$; dans ce cas, le normalisateur du sous-groupe par rapport au groupe B ne peut coïncider aves Γ. Ce normalisateur ne peut être un sous-groupe spécial, car, dans le cas contraire, nous aurions l'égalité

$$T^{-1}P^{\lambda}T=P^{\lambda}$$

c'est-à-dire nous pourrions poser $X=1$.

Ainsi, nous avons démontré que, dans les deux cas envisagés, le groupe B ne peut être simple qu'à la condition de contenir un sous-groupe non spécial d'ordre $p^{\varphi}m$ ($\varphi<\alpha$, m n'est pas divisible par p).

4. Il est connu que l'ordre d'un groupe simple doit être divisible, au moins, par trois nombres premiers différents. L'ordre du groupe B peut, par conséquent, être mis sous la forme $p^{\alpha}q^{\beta}r^{\gamma}u$ (p, q, r — nombres premiers). Nous avons démontré que le groupe B doit posséder un sous-groupe non spécial d'ordre $p^{\varphi}m$. Le nombre m est un diviseur du nombre $q^{\beta}r^{\gamma}u$ (différent de l'unité). Supposons que m est divisible par q.

Considérons, en premier lieu, le cas où m n'est pas divisible par r. En raisonnant comme nous l'avons fait précédemment, nous pouvons démontrer que le groupe B doit contenir un sous-groupe non spécial d'un ordre divisible par une certaine puissance du nombre r. Ce sous-groupe non spécial ne peut être conjugué au sous-groupe non spécial d'ordre $p^{\varphi}m$ (les ordres des sous-groupes conjugués doivent être égaux). Nous voyons ainsi que, contrairement à l'hypothèse, le groupe B contient au moins deux classes de sous-groupes non spéciaux. Envisageons, maintenant, le cas où m est divisible par r. L'ordre $p^{\varphi}m$ du sous-groupe non spécial considéré doit alors être divisible par trois nombres premiers différents. Par conséquent (d'après le résultat cité précédemment de O. Schmidt), ce sous-groupe doit contenir au moins un sous-groupe non spécial. Nous arrivons également à la conclusion que le groupe B possède au moins deux classes de sous-groupes non spéciaux.

Ainsi, nous avons démontré le théorème suivant:

Un groupe, qui ne possède qu'une classe de sous-groupes non spéciaux, est résoluble.

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MATHÉMATIQUES

UN THÉORÈME SUR LES COMPLEXES SIMPLICIAUX (1)

Par M. BEBUTOV

(Présenté par S. N. Bernstein, de l'Académie, le 17. III. 1938)

Je me propose de démontrer, dans cette note, le théorème suivant donnant la réponse positive à une question posée par P. Alexandroff.

Théorème. *Les seuls cas, où un complexe simplicial K à n dimensions admet un complexe simplicial réciproque, sont les cas suivants:*

pour $n=1$: *le complexe K est un polygone simple fermé ou la réunion de plusieurs polygones simples fermés, deux à deux disjoints;*

pour $n>1$: *le complexe K est constitué d'un ou de plusieurs complexes disjoints, isomorphes à la frontière d'un simplexe à $n+1$ dimensions.*

D'ailleurs dans les deux cas, l'un des deux complexes réciproques est isomorphe à l'autre.

Dans l'énoncé précédent la réciprocité des complexes est comprise dans le sens classique (Poincaré): deux complexes K et L sont dits réciproques, s'il existe une correspondance biunivoque f entre les éléments de K et de L telle que la condition suivante soit vérifiée: si le simplexe x' est une face du simplexe x dans l'un des deux complexes, le simplexe $f(x)$ est une face du simplexe $f(x')$ dans l'autre.

Remarquons enfin qu'on peut donner au théorème mentionné l'énoncé suivant:

Sauf dans les deux cas indiqués, un espace discret et son espace réciproque ne peuvent jamais être tous deux isomorphes à des complexes simpliciaux (2).

Démonstration. Seul le cas $n>1$ exige une discussion. Nous désignerons par K et L deux complexes simpliciaux réciproques, en réservant les lettres x et y pour des éléments quelconques de ces complexes. L'élément correspondant par réciprocité à l'élément donné x sera désigné par $f(x)$. Nous supposons les deux complexes connexes (autrement on examinerait les composantes de ces complexes). Il s'agit donc de prouver que, sous les conditions énoncées, les deux complexes se trouvent isomorphes à la frontière d'un simplexe.

On s'aperçoit sans peine des faits suivants:

1) les deux complexes ont la même dimension;

2) ils sont dimensionnellement homogènes;

3) l'indice supérieur désignant la dimension d'un élément, on a toujours: $f(x^r)=y^{n-r}$.

Cela posé, le nombre des simplexes à s dimensions, contenant un x^r comme face, est égal au nombre des faces $(n-s)$-dimensionnelles d'un

simplexe $(n-r)$-dimensionnel (à savoir du simplexe $f(x^r)$, c'est-à-dire à $\binom{n-r+1}{n-s+1}$). Il en résulte en particulier qu'on a, dans l'un quelconque des deux complexes, $n+1$ simplexes à n dimensions contenant un sommet donné et n simplexes à n dimensions contenant une arête donnée. De même, il y a $n+1$ arêtes aboutissant à un sommet donné.

Démontrons maintenant que *deux sommets arbitraires x_1^0 et x_2^0 appartiennent à une même arête.* Les complexes donnés étant connexes, on réduit aisément le cas général au cas, où il existe des arêtes adjacentes $x_1^1=(x_1^0 x_3^0)$ et $x_2^1=(x_3^0 x_2^0)$. Or, l'arête x_1^1 appartient à n simplexes n-dimensionnels, et il est de même pour l'arête x_2^1. Les uns et les autres de ces simplexes contiennent le sommet x_3^0. Comme il n'y a que $n+1$ simplexes n-dimensionnels contenant un sommet donné, et comme pour $n>1$ le nombre $2n$ est supérieur à $n+1$, l'un au moins des simplexes contenant x_1^1 doit contenir x_2^1, donc les trois sommets x_1^0, x_2^0, x_3^0. Notre assertion se trouve ainsi démontrée.

Choisissons, dans l'un de nos complexes, un sommet x^0. Comme il existe, pour tout sommet x_1^0, différent de x^0, une arête $(x^0 x_1^0)$ et le nombre total des arêtes aboutissant à x^0 est égal à $n+1$, *le nombre des sommets dans chacun des complexes K et L est égal à $n+2$.* Par raison de réciprocité, *le nombre des simplexes n-dimensionnels dans chacun de ces complexes est aussi égal à $n+2$.* Reste à montrer qu'on peut trouver, par exemple, dans K, un simplexe contenant r sommets arbitraires de ce complexe, sous la seule condition qu'on ait $r\leqslant n+1$. Soient en effet $x_1^0,\ldots,x_r^0$ les r sommets donnés. Si ces sommets n'appartiennent à aucun simplexe x^n de K, c'est qu'il existe pour tout x^n un sommet parmi les $x_1^0,\ldots,x_r^0$ qui ne lui appartient pas. Or, le nombre des x^n étant $n+2$, tandis que $r\leqslant n+1$, au moins deux simplexes x^n doivent ne pas contenir le même sommet; mais alors ces simplexes sont nécessairement identiques. La contradiction ainsi obtenue achève la démonstration.

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MATHÉMATIQUES

DÉFORMATION D'UNE CONGRUENCE RECTILIGNE AVEC CONSERVATION DES SURFACES DISTRIBUTIVES

Par S. BOSSINSKI

(Présenté par S. A. Tchapliguine, de l'Académie, le 20. III. 1938)

§ 1. On appelle surfaces distributives d'une congruence rectiligne celles de ses surfaces réglées dont les lignes de striction se trouvent sur la surface moyenne de la congruence.

Le problème général sur la déformation d'une congruence rectiligne avec conservation des surfaces distributives se pose comme il suit.

Supposons que les rayons d'une congruence C sont invariablement liés aux plans tangents correspondants d'une surface quelconque S. Pour quelles surfaces S et pour quelles congruences C les rayons de C formant les suites des génératrices des surfaces distributives de C restent-ils toujours des génératrices des surfaces distributives de la congruence C (la surface S au cours de la déformation soit continue et d'ailleurs arbitraire, soit à réseau conjugué persistant)?

§ 2. Dans cette noté nous étudions les congruences C dont les rayons sont orthogonaux aux plans tangents correspondants de la surface S.

Dans le cas d'une déformation arbitraire de S on n'arrive qu'aux surfaces développables S; c'est un cas privé d'intérêt.

Supposons maintenant que la surface S peut se déformer avec conservation d'un réseau conjugué, c'est-à-dire qu'elle est douée d'une base principale, et rapportons-la à cette base.

Désignons par $\bar{\rho}(u, v)$, $\bar{n}(u, v)$ le vecteur définissant la surface S et le vecteur unitaire de sa normale. Soient $E, F, G, D, D' = 0, D''$ les coefficients des formes $d\bar{\rho}^2$ et $-d\bar{\rho}\,d\bar{n}$ attachées à S.

Le pied du rayon de la congruence C sur le plan tangent correspondant de S définissons par le vecteur:

$$\bar{\rho}_1(u, v) = \bar{\rho} + \xi(u, v)\,\bar{\rho}_u + \eta(u, v)\,\bar{\rho}_v.$$

Si l'on choisit pour surface de départ de la congruence C la surface définie par le vecteur $\bar{\rho}_1(u, v)$, on trouve pour les coefficients $E_0, F_0, G_0, e, f, f', g$ des formes fondamentales de Kummer les expressions suivantes:

$$\left.\begin{aligned} &E_0 = \bar{n}_u^2 = \frac{GD^2}{H^2}, \quad F_0 = \bar{n}_u\bar{n}_v = -KF, \quad G_0 = \bar{n}_v^2 = \frac{ED''^2}{H^2}, \\ &e = \bar{n}_u\bar{\rho}_{1u} = \frac{Fe_2 - Ge_1}{H^2}D, \quad f = \bar{n}_u\bar{\rho}_{1v} = \frac{Fg_2 - Gg_1}{H^2}D, \\ &f' = \bar{n}_v\bar{\rho}_{1u} = \frac{Fe_1 - Ee_2}{H^2}D'', \quad g = \bar{n}_v\bar{\rho}_{1v} = \frac{Fg_1 - Eg_2}{H^2}D'', \end{aligned}\right\} \qquad (1)$$

K étant la courbure totale de la surface S, $H^2 = EG - F^2$ et où l'on a posé:

$$e_1 = E\left[1 + \xi_u + \xi\left\{\begin{matrix}11\\1\end{matrix}\right\} + \eta\left\{\begin{matrix}12\\1\end{matrix}\right\}\right] + F\left[\eta_u + \xi\left\{\begin{matrix}11\\2\end{matrix}\right\} + \eta\left\{\begin{matrix}12\\2\end{matrix}\right\}\right],$$

$$e_2 = F\left[1 + \xi_u + \xi\left\{\begin{matrix}11\\1\end{matrix}\right\} + \eta\left\{\begin{matrix}12\\1\end{matrix}\right\}\right] + G\left[\eta_u + \xi\left\{\begin{matrix}11\\2\end{matrix}\right\} + \eta\left\{\begin{matrix}12\\2\end{matrix}\right\}\right],$$

g_2 et g_1 se déduisant de e_1 et e_2 respectivement à l'aide des substitutions (u, v), (E, G), (ξ, η).

L'équation différentielle des surfaces distributives de C est

$$\begin{aligned}&[E_0(E_0 g - G_0 e) - F_0(E_0 f + E_0 f' - 2F_0 e)]\,du^2 + \\ &+ 2[F_0(E_0 g + G_0 e) - E_0 G_0(f + f')]\,du\,dv + \\ &+ [G_0(G_0 e - E_0 g) - F_0(G_0 f + G_0 f' - 2F_0 g)]\,dv^2 = 0;\end{aligned}$$

en vertu de l'équation de Gauss $DD'' = KH^2$ elle prend la forme suivante:

$$D^6 \lambda\, du^2 + D^4(\lambda_1 du^2 + 2\mu_1 du\, dv) + D^2(2\mu_2 du\, dv + \nu_2 dv^2) + \nu_3 dv^2 = 0,$$

où, le cas banal des surfaces développables ($K = 0$) étant rejeté, on a:

$$\left.\begin{aligned}&\lambda = -Gg_2, && \mu_2 = (KH^2)^2 Ee_2,\\ &\lambda_1 = KH^2(Ge_1 - 2Fe_2), && \nu_2 = (KH^2)^2(Eg_2 - 2Fg_1),\\ &\mu_1 = KH^2 Gg_1, && \nu_3 = -(KH^2)^3 Ee_1.\end{aligned}\right\} \qquad (2)$$

Le problème posé donne lieu aux quatre équations suivantes:

$$\left.\begin{aligned}&\lambda du^2 = 0, \quad \lambda_1 du^2 + 2\mu_1 du dv = 0,\\ &2\mu_2 du dv + \nu_2 dv^2 = 0, \quad \nu_3 dv^2 = 0,\end{aligned}\right\} \qquad (3)$$

qui doivent évidemment se réduire à une seule.

Le cas dépourvu d'intérêt des surfaces distributives confondues étant rejeté, on arrive tout d'abord aux conditions suivantes:

$$\lambda = 0, \quad \nu_3 = 0$$

ou bien, en tenant compte des formules (2), aux conditions:

$$Gg_2 = 0, \quad Ee_1 = 0.$$

§ 3. Il est nécessaire, par conséquent, à examiner les quatre hypothèses suivantes:

1) Soient $E = G = 0$ et, par suite, $\mu_1 = \mu_2 = 0$. Les équations (3) montrent que l'on a aussi:

$$\lambda_1 = \nu_2 = 0.$$

On voit des formules (2) que cela entraîne les équations:

$$e_2 = 0, \quad g_1 = 0. \qquad (4)$$

Par conséquent, la congruence C doit être une congruence isotrope et la surface S correspondante une surface minima arbitraire.

Les équations (4) définissent le pied du rayon de la congruence C sur le plan tangent correspondant de la surface S.

C'est une solution banale du problème posé, car les surfaces distributives d'une congruence isotrope sont indéterminées.

2) Soient maintenant

$$e_1 = 0, \quad g_2 = 0. \qquad (5)$$

Les deux équations (3) qui restent dans ce cas auront la forme:

$$Fe_2du^2 - Gg_1dudv = 0, \quad Ee_2dudv - Fg_1dv^2 = 0,$$

et comme elles doivent évidemment se réduire à l'équation $dudv = 0$, on a:

$$Fe_2 = 0, \quad Fg_1 = 0.$$

L'hypothèse $e_2 = 0$, $g_1 = 0$ conduit seulement aux surfaces développables S (1). Par suite, supposons que

$$F = 0.$$

Dans ce cas la surface S se déforme avec conservation des lignes de courbure (u, v) et, par conséquent, se présente, en tenant compte des résultats bien connus de Bonnet et Codazzi, comme une surface moulure de Monge.

La congruence correspondante C n'est pas une congruence de normales comme il résulte des formules (1). Ses surfaces distributives correspondent aux lignes de courbure de la surface moulure de Monge. Les équations (5) définissent le pied du rayon de C sur le plan tangent de la surface S.

Ensuite, comme dans le cas considéré, on a l'équation suivante:

$$E_0g - F_0(f + f') + G_0e = 0,$$

la surface de départ de la congruence C coïncide avec la surface moyenne. Cette surface moyenne est le lieu du pied du rayon de C sur le plan tangent correspondant de la surface moulure de Monge.

Il en suit que la déformation considérée conserve la surface moyenne de la congruence C.

On voit d'ailleurs que la surface moulure indiquée se présente comme l'enveloppée moyenne de la congruence C et conserve la propriété d'être l'enveloppée moyenne de C au cours de sa déformation à base principale.

Ce résultat est une conséquence d'un théorème général établi récemment par l'auteur (2) sur la conservation de l'enveloppée moyenne d'une congruence au cours de sa déformation à base principale.

Enfin, en vertu des formules (1), il est aisé de voir que les foyers de la congruence C restent invariables tandis que les points-limites de chaque rayon de C se déplacent pendant la déformation.

3. Il reste encore à examiner les hypothèses $G = 0$, $e_1 = 0$ et $E = 0$, $g_2 = 0$. Toutes les deux conduisent seulement aux surfaces développables S (1).

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MATHÉMATIQUES

L'EXTENSION DE L'OPÉRATION DE L'INTÉGRATION SINGULIÈRE POUR L'ESPACE L_2

Par S. MICHLIN

(Présenté par I. M. Vinogradow, de l'Académie, le 19. III. 1938)

Nous avons obtenu (1) des formules pour la composition des intégrales singulières doubles. Ces formules nous ont permis d'établir (2) que l'opération de l'intégration singulière est linéaire dans l'espace L_2. En généralisant nos résultats, G. Giraud (3) a obtenu des formules pour la composition des intégrales singulières multiples. En utilisant ces formules de G. Giraud, nous démontrons le théorème suivant:

Les intégrales singulières multiples sont des opérations linéaires dans l'espace L_2. On peut les développer en séries suivant les puissances de certaines opérations unitaires.

Considérons l'espace L_2 des fonctions $W(M)$, detérminées presque partout dans l'espace euclidien E des coordonnées $x_1, x_2, \dots x_m$ *, et satisfaisant aux conditions suivantes: 1) le produit

$$[1+(x_1^2+x_2^2+\dots+x_m^2)^{\frac{m}{2}}]\,W(M) \tag{1}$$

est borné à l'infini, et 2) $|W(M)|^2$ est sommable dans E. Nous appelons comme d'habitude norme de $M(W)$ la quantité $\left\{\int\limits_E |W(M)|^2 d\tau\right\}^{\frac{1}{2}}$. Soient M et M_1 deux points de E. Désignons par R, $\theta_1, \dots \theta_{m-2}, \varphi$ les coordonnées sphériques du point M_1, liées au pôle M. Ici R désigne la distance entre M et M_1; les angles $\theta_1, \dots \theta_{m-2}, \varphi$ satisfont aux inégalités

$$0 \leqslant \theta_\alpha \leqslant \pi, \quad -\pi \leqslant \varphi \leqslant \pi.$$

Remarquons que les coordonnées sphériques du point M liées au pôle M_1 sont R, $\pi-\theta_1, \dots \pi-\theta_{m-2}$, $\pi+\varphi$.

L'intégrale

$$TW(M) = \int\limits_E W(M_1)\,\frac{f(M;\,\theta_1, \dots \theta_{m-2}, \varphi)}{R^m}\,d\tau_1, \tag{2}$$

prise au sens de sa valeur principale, existe, si la fonction (1) satisfait à la condition de Hölder ** et si l'on a

* $M(x_1, x_2, \dots x_m)$ est un point de E.

** Nous appelons «fonctions de la classe H» les fonctions satisfaisant à cette condition.

$$\int f(M; \theta_1, \dots \theta_{m-2}, \varphi)\, d\omega = 0. \tag{3}$$

Ici S désigne l'hypersphère de centre M et de rayon 1. Nous supposons encore que f est bornée et satisfait à la condition de Lipschitz. Si f ne dépend pas de M, les opérations (2) sont commutatives. Dans le cas général $T_1T_2 - T_2T_1$ est une opération totalement continue dans L_2. L'opération (2) fait correspondre à chaque fonction de la classe H une fonction de la même classe (4).

Développons $f(M; \theta_1, \dots \theta_{m-2}, \varphi)$ en une série suivant les fonctions hypersphériques à m dimensions:

$$f(M; \theta_1, \dots \theta_{m-2}, \varphi) = \sum_{n=1}^{\infty} P_{n,m}(M; \theta_1, \dots \theta_{m-2}, \varphi). \tag{4}$$

D'après les formules mentionnées de G. Giraud à chaque opération

$$a(M)W(M) + \int_E W(M_1) \frac{f(M; \theta_1, \dots \theta_{m-2}, \varphi)}{R^m}\, d\tau_1 \tag{5}$$

on fait correspondre la fonction

$$F = a(M) + \sum_{n=1}^{\infty} \mu_n P_{n,m}(M; \theta_1, \dots \theta_{m-2}, \varphi);\ \mu_n = \frac{i^n \pi^{\frac{m}{2}}\, \Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{m+n}{2}\right)}. \tag{6}$$

Nous appelons cette fonction le symbole * de l'opération (5). Il correspond à la somme et au produit des opérations (5) respectivement la somme et le produit des symboles. Nous écrirons l'opération (5) comme $F \star W(M)$. Le système des fonctions

$$e^{2ik_1\theta_1 + 2ik_2\theta_2 + \dots + 2ik_{m-2}\theta_{m-2} + ik\varphi}$$

est orthogonal et complet dans le parallélépipède

$$0 \leqslant \theta_\alpha \leqslant \pi,\ -\pi \leqslant \varphi \leqslant \pi.$$

Il en suit qu'on peut développer l'opération (5) en une série suivant les puissances des opérations dont les symboles sont

$$e^{i\varphi},\ e^{2i\theta_\alpha},\ \alpha = 1, \dots m-2. \tag{7}$$

Démontrons que les opérations qui correspondent aux symboles (7) *sont unitaires dans* L_2.

Il suffit de le démontrer pour les fonctions de la classe H, parce que l'ensemble de ces fonctions est dense dans L_2.

Le symbole $e^{2i\theta_\alpha}$ correspond à une certaine opération

$$a_\alpha W(M) + \int_E W(M_1) \frac{f_\alpha}{R^m}\, d\tau_1 = e^{2i\theta_\alpha} \star W(M) = V(M). \tag{8}$$

Développons f_α en une série suivant les fonctions hypersphériques

$$f_\alpha = \sum_{n=1}^{\infty} Q_n^{(\alpha)}(\theta_1, \dots \theta_{m-2}, \varphi). \tag{9}$$

Alors

$$e^{2i\theta_\alpha} = a_\alpha + \sum_{n=1}^{\infty} \mu_n Q_n^{(\alpha)}(\theta_1, \dots \theta_{m-2}, \varphi). \tag{10}$$

* Comparer avec (1).

On peut démontrer que le nombre a_α est réel, ainsi que les fonctions $Q_n^{(\alpha)}$, et

$$Q_n^{(\alpha)}(\pi-\theta_1,\dots\pi-\theta_{m-2},\pi+\varphi)=(-1)^n Q_n^{(\alpha)}(\theta_1,\dots\theta_{m-2},\varphi).$$

Nous avons

$$I=\int_E |V(M)|^2\,d\tau=\int_E \overline{V(M)}\,d\tau\Big[a_\alpha W(M)+$$

$$+\int_E\sum_{n=1}^{\infty}Q_n^{(\alpha)}(\theta_1,\dots\theta_{m-2},\varphi)\frac{W(M_1)}{R^m}\,d\tau_1\Big]=$$

$$=a_\alpha\int_E\overline{V(M)}\,W(M)\,d\tau+\int_E W(M_1)\,d\tau_1\int_E\sum_{n=1}^{\infty}Q_n^{(\alpha)}(\theta_1,\dots\theta_{m-2},\varphi)\frac{\overline{V(M)}}{R^m}\,d\tau=$$

$$=\int_E W(M)\,d\tau\Big[a_\alpha\overline{V(M)}+\int_E\sum_{n=1}^{\infty}(-1)^n Q_n^{(\alpha)}(\theta_1,\dots\theta_{m-2},\varphi)\frac{\overline{V(M_1)}}{R^m}d\tau_1\Big]=$$

$$=\int_E W(M)\,[e^{-2i\theta_\alpha}\star\overline{V(M)}]\,d\tau.$$

Il est aisé de voir que

$$\overline{V(M)}=e^{2i\theta_\alpha}\star\overline{W(M)}.$$

Il en suit directement que

$$I=\int_E |W(M)|^2\,d\tau. \tag{11}$$

Notre affirmation est ainsi démontrée. Des raisonnements analogues permettent de démontrer la même affirmation pour l'opération dont le symbole est égal à $e^{i\varphi}$.

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MATHÉMATIQUES

QUELQUES THÉORÈMES NON-LOCAUX SUR LES INTÉGRALES SINGULIÈRES

Par I. NATANSON

(Présenté par S. N. Bernstein, de l'Académie, le 25. III. 1938)

I. Théorème I. *Si le noyau* $\Phi_n(t, x)$ *de l'intégrale singulière*

$$f_n(x) = \int_a^b f(t)\,\Phi_n(t, x)\,dt \qquad (1)$$

satisfait aux conditions suivantes

a) $$\lim_{n\to\infty} \int_\alpha^\beta \Phi_n(t, x)\,dt = 1 \qquad (\alpha < x < \beta),$$

b) $$\int_a^b |\Phi_n(t, x)|\,dt < K,$$

c) $$\int_a^b |\Phi_n(t, x)|\,dx < K,$$

alors, quelle que soit la fonction $f(x) \in (L^p)$, $(p \geqslant 1)$, *nous avons:*

$$\lim_{n\to\infty} \int_a^b |f_n(x) - f(x)|^p\,dx = 0. \qquad (2)$$

Démonstration. Si l'intervalle (a, b) peut être divisé en un nombre fini de segments dans chacun desquels la fonction $f(x)$ est constante, le théorème est évident. En effet, dans ce cas nous pouvons passer à la limite sous le signe d'intégrale, parce que les fonctions $|f_n(x) - f(x)|$ sont uniformément bornées et tendent vers zéro en tous les points intérieurs aux intervalles, où $f(x)$ est constante.

Mais la classe des fonctions de cette nature est dense dans l'espace (L^p). Alors, comme il a été demontré par W. Orlicz [1], la relation (2) a lieu quelle que soit la fonction $f(x) \in (L^p)$.

Remarque. Nous pouvons remplacer la condition a) par la condition a*) plus large:

a*) L'intégrale $\int_a^\beta \Phi_n(t, x)\,dt$ converge en mesure vers l'unité dans l'intervalle $\alpha \leqslant x \leqslant \beta$.

2. Théorème II. *Si le noyau* $\Phi_n(t, x)$ *satisfait aux conditions suivantes:*

a) $\lim_{n\to\infty} \int_a^\beta \Phi_n(t, x)\,dt = 1 \qquad (\alpha < x < \beta)$,

d) *Il existe une fonction majorante* $\Psi_n(t, x)$ *telle que*

1) $|\Phi_n(t, x)| \leqslant \Psi_n(t, x)$.

2) *La fonction* $\Psi_n(t, x)$ *(comme la fonction de t seul) est croissante dans l'intervalle* (a, x) *et décroissante dans l'intervalle* (x, b).

3) $\int_a^b \Psi_n(t, x)\,dt < K$,

alors, quelle que soit la fonction sommable $f(x)$, *nous avons presque partout*

$$\lim f_n(x) = f(x) \qquad (3)$$

et d'ailleurs si $f(x) \in (L^p)$, $(p > 1)$, *alors il existe une fonction* $\theta(x) \in (L^p)$, *telle que*

$$|f_n(x)| \leqslant \theta(x).$$

En particulier la relation (2) *a lieu.*

Démonstration. La relation (3) est une conséquence d'un théorème de D. Faddeeff (2). D'autre part j'ai démontré (3) l'inégalité

$$|f_n(x)| \leqslant 4KM(x),$$

où

$$M(x) = \max_{\alpha \leqslant x \leqslant \beta} \left[\frac{1}{\beta - \alpha} \int_a^\beta |f(t)|\,dt\right].$$

Or, G. H. Hardy et I. E. Littlewood (4) ont démontré que

$$M(x) \in (L^p) \qquad (p > 1)$$

chaque fois que $f(x) \in (L^p)$. Notre théorème est ainsi démontré.

Remarque 1. Dans le cas $p = 1$ le théorème est faux, parce que la fonction $M(x)$ peut être non-sommable. Mais si l'intégrale

$$\int_a^b |f(x)| \ln^+ |f(x)|\,dx$$

est finie, alors

$$\lim_{n\to\infty} \int_a^b |f_n(x) - f(x)|\,dx = 0.$$

Remarque 2. Tous les deux théorèmes sont applicables aux intégrales suivantes:

$$W_n(x) = \sqrt{\frac{n}{\pi}} \int_a^b f(t)\, e^{-n(t-x)^2}\, dt,$$

$$F_n(x) = \frac{1}{2n\pi} \int_{-\pi}^{\pi} f(t) \left[\frac{\sin n \frac{t-x}{2}}{\sin \frac{t-x}{2}} \right]^2 dt,$$

$$P_r(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, \frac{1-r^2}{1-2r\cos(t-x)+r^2}\, dt,$$

$$B_n(x) = \frac{S_n(x) + S_n\left(x + \frac{2\pi}{2n+1}\right)}{2},$$

où $S_n(x)$ est la somme des n termes de la série de Fourier de la fonction $f(x)$:

$$L_n(x) = \sqrt{\frac{n}{\pi}} \int_0^1 f(t)\,[1-(t-x)^2]^n\, dt,$$

$$V_n(x) = \frac{\sqrt{n}}{2\pi} \int_{-\pi}^{\pi} f(t) \cos^{2n} \frac{t-x}{2}\, dt,$$

$$K_n(x) = (n+1) \sum_{k=0}^{n} C_n^k\, x^k (1-x)^{n-k} \int_{\frac{k}{n+1}}^{\frac{k+1}{n+1}} f(t)\, dt,$$

$$S_h(x) = \frac{1}{2h} \int_{x-h}^{x+h} f(t)\, dt.$$

Le théorème I est applicable encore à chaque système de fonctions orthogonales $\{ \omega_k(x) \}$ complet et tel que

$$\int_a^b \left| \sum_{k=1}^{n} \omega_k(x)\, \omega_k(t) \right| dt < K.$$

3. Si le noyau $\Phi_n(t, x)$ pour n fixe est une fonction continue, alors l'opération linéaire

$$U_n(f) = f_n(x)$$

est totalement continue dans l'espace (L^p) (c'est-à-dire elle transforme tout ensemble borné de cet espace en un ensemble compact). Selon une remarque de S. Mazur [5] nous obtenons le résultat suivant:

Théorème III. *Supposons que l'intégrale* (1) *possède un noyau continu satisfaisant aux conditions de l'un des théorèmes* I *ou* II. *La condition nécessaire et suffisante pour qu'un ensemble borné* (A) *soit compact dans l'espace* (L^p) *est que la convergence* (2) *sur* (A) *soit uniforme.*

Si dans ce cas nous utilisons le théorème I, nous obtenons le critérium pour $p \geqslant 1$, mais si nous opérons avec le théorème II, alors notre critérium est applicable seulement dans le cas $p > 1$.

Le théorème III est applicable aux intégrales $W_n(x)$, $F_n(x)$, $P_r(x)$, $B_n(x)$, $L_n(x)$, $V_n(x)$ dans le cas $p \geqslant 1$. Une légère complication de raisonnement nous permet d'appliquer le théorème aussi aux intégrales $K_n(x)$ (pour $p \geqslant 1$) et $S_h(x)$ (mais seulement pour $p > 1$, cependant, comme il a été démontré par A. Tulajkoff [6], le résultat reste vrai aussi pour $p = 1$).

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MATHEMATICS

A CLASSIFICATION OF CONTINUOUS TRANSFORMATIONS OF A COMPLEX INTO A SPHERE. 2*

By L. PONTRYAGIN

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In this paper we give a classification of transformations of the sphere S_{n+2} into the sphere S_n $(n \geqslant 2)$ and establish some general relations between transformations of S_r into S_n.

Theorem 1. *There exist exactly two classes of transformations of* S_4 into S_2.

Proof. In C. of c. t. 1 it was proved that the number of classes of transformations of S_4 into S_2 is equal to the number of classes of transformations of S_4 into S_3. In the same paper this last number was proved to be 2.

Theorem 2. If $n \geqslant 3$ there exists only one class of transformations of S_{n+2} into S_n.

Proof. According to C. of c. t. 1 we may confine ourselves to analytical transformations of S_{n+2} into S_n. Let f be an analytical transformation of S_{n+2} into S_n and a a point of S_n, such that $f^{-1}(a)$ is a surface in S_{n+2} without singularities. Let further V be a (sufficiently small) spherical neighbourhood of a. Then $U = f^{-1}(V)$ is homeomorphic to the topological product of the full sphere V and a given surface P; moreover, if a point $z \in U$ corresponds (in this homeomorphism) to (x, y) (where $x \in V$ and $y \in P$), then $f(z) = f(x, y) = x$. We can modify f so that it will transform the full sphere (V, y) into the full sphere V isometrically and that (V, y) will be orthogonal to the surface (a, P) for an arbitrary $y \in P$.

We shall first consider the case when P is a sphere. For $n \geqslant 4$ any two differentiable surfaces homeomorphic to a sphere are homotopic in S_{n+2}. If, therefore, f and g are two transformations of the described type, we can suppose that the surface (a, P), and consequently, the full spheres

* The present paper is a continuation of my paper «A classific tion of continuous transformations of a complex into a sphere. 1» [C. R. Acad. Sci. URSS, XIX, № 1—2 (1938)], which will be referred to as C. of c. t. 1.

The convention laid down in C. of c. t. 1 concerning the distinction we make between a k-dimensional «sphere» and a k-dimensional «full sphere» remains valid: under a k-dimensional sphere we understand the boundary of a $(k+1)$-dimensional full sphere.

Theorems 1 and 2 were reported on my behalf by Prof. S. Lefshetz at the International Mathematics Congress in Oslo in 1936.

(V, y) coincide for both transformations. We have thus $f(x, y) = g(\psi_y(x), y)$ where ψ_y is a rotation of V depending in a continuous manner on the point $y \in P$. Any image of a two-dimensional sphere being homotopic to zero in the manifold of all rotations of V, we conclude that f and g are equivalent (see C. of c. t. 1, A). The case $n=3$ requires a special consideration (involving some rather complicated arguments), but the result remains true.

It remains to reduce the general case to the case when P is a sphere. It is easily seen that the surface P is always orientable. Let L be a simple closed curve lying in P. A neighbourhood of L can be covered by a monoparametric family $\{L_t\}$, $-1 < t < 1$, of simple closed curves $(L_0 = L)$. Consider in S_{n+2} the family of curves $\{(x, L_t)\}_{x \in V}$. This family is homeomorphic to an $(n+1)$-dimensional full sphere V'. If we now correlate every curve (x, L_t) with the point of V' which corresponds to the curve (x, L_t) in the above homeomorphism, we shall obtain a transformation h' of an open subset of S_{n+2} into V'. Supposing that V' lies in an $(n+1)$-dimensional sphere S_{n+1}, we can extend the transformation h' to a transformation h of S_{n+2} into S_{n+1} (see C. of c. t. 1, B). Two cases are now possible: 1) the transformation h is homotopic to zero; then we shall say that the index of L is zero; 2) h is not homotopic to zero; in this case we shall say that the index of L is unity. It may be proved that homologous curves on P have the same index and that if we add two curves (in the sense of homology) their indices are also added modulo 2. Hence it follows easily that unless P is a system of spheres, there exists on P a curve not homologous to zero, whose index is zero. Let L be such a curve. Denote by P' an differentiable surface in S_{n+2}, homeomorphic to a circle (two-dimensional full sphere), having the curve (a, L) for its boundary and no points in common with the surface (a, P) except the points of (a, L).

This construction enables us to modify f so as to make the genus of P smaller by one. Repeating this process we shall come at last to a transformation for which the corresponding surface P consists of a system of spheres; and a system of spheres may be easily reduced to a single sphere.

The theorem is thus proved.

It seems to be usefull to introduce the operations of addition of classes of equivalent transformations of S_r into S_n, thus transforming the set of all such classes into a group.

Definition. Let S_r' and S_r'' be two oriented r-dimensional spheres, and f' and f'' their transformations into the n-dimensional sphere S_n. We shall say that the transformations f' and f'' are equivalent, if there exists a homeomorphism h of S_r'' on S_r' which preserves the orientation and for which the transformation $f'h$ and f'' of S_r'' into S_n are equivalent. Thus, all transformations of oriented r-dimensional spheres into S_n are divided into classes of equivalent transformations.

We shall define now the sum of two classes as follows. Let A and B be two classes and let $f \in A$ and $g \in B$. Denote by S_r' the sphere transformed by f and by S_r'' that transformed by g. Let further V' and V'' be two full spheres resp. in S_r' and S_r''; since f and g are defined only up to equivalent transformations, we may suppose that $f(V') = g(V'') = a$ where a is a point of S_n. Now if we join the oriented manifolds $S_r' - V'$ and $S_r'' - V''$ by identifying their boundaries so that the orientations are preserved, we obtain an oriented sphere S_r'''; the transformations f and g of S_r' and S_r'' define a continuous transformation h of S_r''' into S_n.

It may be proved that the class C, to which h belongs, depends neither on the choice of f and g within the classes A and B, nor on the way of identification of the boundaries of $S_r' - V'$ and $S_r'' - V''$ (provided that the condition about the preservation of orientations is fulfilled), but only on the classes A and B. This class C we shall call the sum of A and B and write $C = A + B$. The operation of addition thus defined possesses all the properties of a group operation. The zero of this group is the class of transformations homotopic to zero. If A is a class of transformations and $f \in A$ is a transformation of S_r' into S_n, then the same transformation f applied to the sphere S_r'', differing from S_r' by orientation, determines the class $-A$ opposite to A.

The group thus obtained is isomorphic to one of the homotopy groups constructed by Hurewicz.

The problem of classification of continuous transformations of S_{n+k} into S_n may be now more precisely formulated as the problem of determination of the group of transformations of S_{n+k} into S_n. This group we shall denote by P_k^n.

From the results already ortained it follows that P_1^2 is a free cyclic group; P_1^n for $n \geqslant 3$ is a cyclic group of the second order, as also is P_2^2, while for $n \geqslant 3$ the group P_2^n consists of a single element.

We have no reasons to suppose that P_k^n is always a cyclic group. But another conjecture suggested by the above results is true. We have:

Theorem 3. *For $n \geqslant k+2$ P_k^n is isomorphic to $P_k^{k+2} = P_k$.*

The proof (which is not very complicated) depends upon the propositions D and A of C. of c. t. 1.

In view of this theorem the problem arises to determine the group P_k. It seems that this problem is closely related to the study of homotopic properties of the group of orthogonal matrices.

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AERODYNAMICS

ON PRESSURES DEPENDENT UPON VORTEX-FORMATION CAUSED BY A SUDDEN INCREASE IN THE DENSITY OF THE STREAM

By A. DONOFF

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In the present work an investigation is made of a two-dimensional stationary no heat conducting stream of an ideal gas moving with a velocity exceeding that of sound, which flows past a rigid smooth wall (the washed contour), located in such a way with reference to the stream, that a sudden increase in density is formed in front of it.

Prior to passing the shock-wave the stream is assumed to be constant and parallel to the axis of the abscissae (non-disturbed flow). The magnitudes of velocity, pressure, density and velocity of sound propagation in this stream are respectively denoted by w, p_0, ρ_0, a_0, while ratio $\frac{w}{a_0}$ is symbolized by M. Since a stream exceeding the velocity of the sound is being considered, number M exceeds unity.

After passing the shock-wave the stream becomes, generally speaking, variable and vertical (disturbed flow). We shall study here the following functions of coordinates x and y: v, β, p, ρ which determine the values of velocity, angle of inclination of velocity to axis x, pressure and density in disturbed flow respectively. Functions v, β, p, ρ satisfy the equations of hydrodynamics, the condition of flowing past a rigid smooth wall, as well as the dynamic conditions on the line of the discontinuity. Moreover, these functions, just as the form of the washed contour, are assumed to depend on a certain parameter λ, the dependence on this parameter being of such a nature that with λ equalling zero, the contour washed turns into a segment of the axis of the abscissae, while a disturbed stream turns into a non-disturbed one. Along with the latter condition, several further conditions of a more general nature are laid down, which would warrant the regularity of the functions considered in certain regions into which the total field taken by the turbulent stream may be split up.

Since the stream under consideration is supposed to have a velocity exceeding that of sound, the region occupied by disturbed flow can be covered by two sets of characteristics. As is well known, along any one of the characteristics of one of these sets, called by us the first, the following relations are fulfilled:

$$dy = m_1 dx; \tag{1}$$

$$d(v\cos\beta) + m_2 d(v\sin\beta) = \rho F_1 \frac{d\ln\frac{\vartheta}{\vartheta_0}}{d\psi} dx, \tag{2}$$

At the same time, along any characteristic of the other set, called by us the second one, the following relations are fulfilled:

$$dy = m_2 dx, \tag{3}$$

$$d(v\cos\beta) + m_1 d(v\sin\beta) = \rho F_2 \frac{d\ln\frac{\vartheta}{\vartheta_0}}{d\psi} dx. \tag{4}$$

Here we denote by m_1, m_2 certain functions of v and β; ψ is the function of current, while ϑ and ϑ_0 are determined by the equalities: $\vartheta = \frac{p}{\rho^k}$, $\vartheta_0 = \frac{p_0}{\rho_0^k}$ (k is the index of the adiabate of the gas considered). If the right-hand sides of equation (2) and equation (4) equal zero, these equations may be integrated. The integration of equation (2) results in the following relation:

$$\beta + \varphi = \text{const}, \tag{5}$$

fulfilled along any characteristic of the first set; at the same time the integration of equation (4) results in the relation:

$$\beta - \varphi = \text{const}, \tag{6}$$

fulfilled along any one of the characteristics of the second set. We denote here by φ a certain function of v. Relations (5) and (6) are, as is well known, basic for the theory of two-dimensional, potential, stationary adiabatic motion of an ideal gas, exceeding the velocity of sound.

Since in the case in hand we are, in fact, dealing with vortex-formation in a disturbed stream, the right-hand sides of equations (2) and (4) will, generally speaking, differ from zero and these equations cannot be integrated. Yet, it will be possible to make such combinations of differential equations of characteristics, that approximate results, taking due account of vorticity, can be obtained with their help.

Let us see how this is done. It is readily seen that if to equations (1) and (2) of the first set of characteristics we add the expression for the differential of current function:

$$d\psi = \rho v(\sin\beta\, dx - \cos\beta\, dy) \tag{7}$$

and if from equations (1), (2) and (7) we exclude dx and dy, then, in result, the following equation will be obtained:

$$d(v\cos\beta) + m_2 d(v\sin\beta) = \Phi_1 d\ln\frac{\vartheta}{\vartheta_0}, \tag{8}$$

where Φ_1 denotes a certain function of v and β.

On the other hand, having integral (5) of equation:

$$d(v\cos\beta) + m_2 d(v\sin\beta) = 0, \tag{9}$$

it will be easy to find such an integrating factor L_1 of this equation that upon multiplication by this factor, L_1, it will be reduced to the form:

$$d(\beta + \varphi) = 0. \tag{10}$$

If both parts of equation (8) are now multiplied by L_1, this equation will assume the form:

$$d(\beta + \varphi) = H_1 d\ln\frac{\vartheta}{\vartheta_0}, \tag{11}$$

where H_1 denotes a given function of v and β.

Denoting by H_{10} the value of H_1 with $v=w$, $\beta=0$, equation (11) may now be reduced to the form:

$$d(\beta+\varphi)=H_{10}d\ln\frac{\vartheta}{\vartheta_0}+(H_1-H_{10})\,d\ln\frac{\vartheta}{\vartheta_0}. \tag{12}$$

If λ tends to zero, then term $(H_1-H_{10})\,d\ln\frac{\vartheta}{\vartheta_0}$ will have a higher order of smallness than term $H_{10}d\ln\frac{\vartheta}{\vartheta_0}$. After integration of both parts of equation (12) along the characteristic of the first set from point A lying on the jump line to point B located on the contour flown around, we obtain the relation:

$$\beta_b+\varphi_b-(\beta_a+\varphi_a)=H_{10}\cdot\left(\ln\frac{\vartheta_b}{\vartheta_0}-\ln\frac{\vartheta_a}{\vartheta_0}\right)+\int_{\psi_a}^{\psi_b}(H_1-H_{10})\frac{d\ln\frac{\vartheta}{\vartheta_0}}{d\psi}d\psi, \tag{13}$$

where by β_a, φ_a, ϑ_a, ψ_a are denoted the values of β, φ, ϑ, ψ at point A, while β_b, φ_b, ϑ_b, ψ_b denote the values of the same magnitudes at point B.

Making use of the dynamic conditions along the shock-wave and also utilizing Bernoulli's equation as well as some other conditions, which the functions involved in the investigation will satisfy, we shall be able with the help of relation (13) and a relation analogous to it constructed for the second set of characteristics to provide expressions for the hydrodynamical elements of a stream suitable for an approximate calculation of these elements. Thus, for instance, for pressure on the contour flown around the following expression will be obtained:

$$p=p_0(1+c_1\beta_k+c_2\beta_k^2+c_3\beta_k^3+c_4\beta_{k0}^3+c_5\beta_k^4+c_6\beta_{k0}^4+ \\ +c_7\beta_k\beta_{k0}^3+c_8x(\beta_{k0}^3\beta_{k0}'+S_1\lambda^5), \tag{14}$$

where c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8 denote constant numbers, easily calculated by elementary formulae, if only k and M are given; x denotes the abscissa of a point lying on the contour flown around; β_k, the angle of inclination of velocity to axis x on the contour flown around considered as a function of x and λ; β_{k0} and β_{k0}', the values of this function and its derivative with respect to x at the extreme front point of the contour flown around ($x=0$); S_1, some limited function of x and λ. If at the extreme front point of the contour washed we draw a tangential line to this contour and if we supplement the contour by a rectilinear segment of this tangential line, then, with sufficiently small λ, the phenomenon of vorticity will be eliminated, the general character of circulation remaining virtually the same. Comparing the expression for pressure on such a supplemented contour with formula (14) and denoting pressure caused by vorticity of a disturbed flow Δp, we obtain:

$$\Delta p=p_0(c_8x\beta_{k0}^3\beta_{k0}'+S\lambda^5), \tag{15}$$

where S denotes a certain limited function of x and λ. With λ tending to zero the magnitudes $\beta_{k0}^3\beta_{k0}'$ and λ^4 have the same order of smallness. Accordingly, with small values of λ, the approximate expression for vortice-caused pressure will have the following appearance on the contour washed:

$$\Delta p\approx p_0c_8x\beta_{k0}^3\beta_{k0}'. \tag{16}$$

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AERODYNAMICS

CONICAL MOTION OF IDEAL GAS IN THE CASE OF A SUDDEN INCREASE IN ITS DENSITY (BOOSEMANN'S PROBLEM)

By V. BOOLINSKY

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1. Statement of the Problem

In the present work we consider the stationary space motion of an ideal no heat conducting gas, the following conditions being fulfilled:

1) At the boundary of the region where the motion is being studied there exists a point, which is so located that at any internal point O' of the region considered the projection of velocity upon a tangential plane drawn at point O' to a sphere with its centre at point O and passing through point O' will exceed the local velocity of sound $\left(a=\sqrt{\frac{kp}{\rho}}\right)$.

2) The elements characterizing motion at every point of the region are independent of the distance at which this point lies from point O (introduced in 1).

As an instance of such a motion we may take the case when a stream exceeding the velocity of the sound flows round a cone with its vertex at point O and having as directrix some curve of a special kind (corresponding to the cross-section of some thin wing with a sharp leading edge) and subordinate to the condition that all the generatrixes of this cone shall intersect the direction of the velocity of the oncoming stream at such an angle that modulus $\bar{n}\times\bar{v}$ should exceed the local velocity of sound a ($\bar{n}$ is here the single vector of the corresponding generatrix of the cone).

2. Fundamental Equations

$$\operatorname{curl}\bar{v}\times\bar{v}=\frac{k}{k-1}\,p^{\frac{k-1}{k}}\nabla\vartheta;\quad \operatorname{div}\rho\,\bar{v}=0;\quad \frac{d}{dt}\vartheta=0; \tag{1}$$

$\bar{v}$—denotes velocity; p—pressure, ρ—density; $\bar{k}=\frac{c_p}{c_v}$ is the ratio of specific heats; $\vartheta=\frac{p^{\frac{1}{k}}}{\rho}$ is potential temperature.

If the function of current, ψ, is introduced such as to make

$$\rho v_\theta \sin\theta = \frac{\partial\psi}{\partial\lambda};\ \rho v_\lambda = -\frac{\partial\psi}{\partial\theta}$$

(θ and λ are spherical coordinates), then it will be easy to show that $\vartheta = \vartheta(\psi)$.

Making good use of this, as well as of the relation $a^2 = k\vartheta^k \rho^{k-1}$, system (1) may be reduced to a system of 3 scalar equations. Passing to the variables θ_1 and λ, where $\theta_1 = \cos\theta$, this system may be written as follows:

$$\begin{gathered}(a^2 - v_\theta^2)\frac{\partial v_\theta}{\partial\theta_1} - (a^2 - v_\lambda^2)\frac{\partial v_\lambda}{\partial\lambda} - v_\theta v_\lambda \frac{\partial v_\lambda}{\partial\theta_1} + v_\theta v_\lambda \frac{\partial v_\theta}{\partial\lambda} = \\ = a^2 v_\theta \cos\theta - \sin\theta v_r (v_\theta^2 + v_\lambda^2); \\ v_\lambda \frac{\partial v_r}{\partial\lambda} - v_\theta \frac{\partial v_r}{\partial\theta_1} = \sin\theta (v_\theta^2 + v_\lambda^2); \\ v_\theta \frac{\partial v_\lambda}{\partial\theta_1} + v_\theta \frac{\partial v_\theta}{\partial\lambda} + v_r \frac{\partial v_r}{\partial\lambda} = \sin\theta v_r v_\lambda + \theta_1 v_\theta v_\lambda - \\ - \frac{kp}{k-1} v_\theta \sin\theta \frac{d\ln\vartheta}{d\psi}.\end{gathered} \tag{2}$$

3. Characteristics

Equations of system (2), being equations of a hyperbolic type, possess real characteristics, determinable by formulae:

$$\lambda'_{1,2} = \frac{v_\theta v_\lambda \pm a\sqrt{v_\tau^2 - a^2}}{a^2 - v_\theta^2};\ \lambda'_3 = -\frac{v_\lambda}{v_\theta}. \tag{3}$$

Here $v_\tau^2 = v_\theta^2 + v_\lambda^2$.

4. Relations along Characteristics

$$\begin{gathered}\frac{\lambda'_2 - \lambda'_3}{\lambda'_2 - \lambda'_1}\left(\frac{dv_r}{d\theta_1}\right)_1 - \frac{\lambda'_1 - \lambda'_3}{\lambda'_2 - \lambda'_1}\left(\frac{dv_r}{d\theta_1}\right)_2 = -\frac{v_\tau^2}{v_\theta}; \\ \frac{dv_\theta}{d\theta_1} + \lambda'_{1,2}\frac{dv_\lambda}{d\theta_1} \pm \frac{av_r\sqrt{v_\tau^2 - a^2}}{(a^2 - v_\theta^2)(v_\lambda + v_\theta\lambda'_{1,2})}\frac{dv_r}{d\theta_1} = \\ = (a^2 - v_\theta^2)^{-1}\left\{\pm A\sin\theta\frac{v_r v_\lambda}{v_\theta} \pm A\theta_1 v_\lambda + \sin\theta v_r v_\theta^2 - \right. \\ \left. - a^2 v_\theta \theta_1 \pm \frac{Av_r v_\tau^2 \sin\theta}{v_\theta(v_\lambda + v_\theta\lambda'_{1,2})} - \frac{kp}{k-1}\sin\theta\frac{d\ln\vartheta}{d\psi}\right\},\end{gathered} \tag{4}$$

where $A = a\sqrt{v_\tau^2 - a^2}$.

The signs + and 1 correspond to derivatives along the first characteristic; the signs — and 2, to derivatives along the second characteristic.

In the problem of gas dynamics function ϑ usually makes one of the conditions of the problem.

Thus, system (4) represents a system of 3 equations comprising three unknown values.

5. Conditions at Surface of Sharp Rupture

The values of hydrodynamic elements on the surface of a sharp rupture are connected by the relations:

$$\begin{gathered}\rho_+ N_+ \overline{V}_+ - \rho_- N_- \overline{V}_- = p_+\overline{n} - p_-\overline{n}; \\ \rho_+ N_+ - \rho_- N_- = 0; \\ \frac{p_+}{p_-} = \frac{(k+1)\rho_+ - (k-1)\rho_-}{(k+1)\rho_- - (k-1)\rho_+}.\end{gathered} \tag{5}$$

The signs + and — denote hydrodynamic elements before and after passing the surface of sharp rupture; N is the velocity of propagation of the surface of a sharp rupture within a moving medium; $\bar{n}$ is a single vector of an external normal to the surface of rupture.

Hence it follows, primarily, that the radial component of velocity fails to change in passing through a surface of sharp rupture $V_{r+} = V_{r-}$ and secondly that

$$v_{0+}^2 = (v_{\tau-} - v_{\lambda+})^2 \frac{\frac{2}{k+1}\left(v_{\tau-} - \frac{a^2}{v_{\tau-}}\right) - (v_{\tau-} - v_{\lambda+})}{v_{\tau-} - v_{\lambda+} + \frac{2}{k+1}\frac{a^2}{v_{\tau-}}}. \qquad (6)$$

6. Flow round a Wing

Flow round a wing of special form represents that case of «conical» motion which possesses practical interest, wherefore a few brief suggestions for the solution of this problem will be given here. Its distinction from the well-studied plane problem of circulation round a wing consists in the presence of all the three velocity components, though this involves no difficulty in the numerical solution of the problem. Usually, part of the wing profile adjacent to the sharp edge is formed by plane surfaces. In this case the surfaces of sharp rupture incipient at the leading edge of the wing will also be plane along a certain length. Indeed, if we consider flow round an infinite wedge (and this corresponds to the beginning of the wing) and if we further consider this flow relatively to a uniform forward motion of a system of coordinates, then, for this system, given a certain velocity of its motion, the problem will be obviously reduced to that of a stream flowing round the wedge perpendicular to its edge.

The direction of the planes of a sharp rupture are determined as the direction of such planes after passing over which the stream becomes parallel to the plane portion of the wing. After the portion of the surface of sharp rupture with values v_{0+}, $v_{\lambda+}$, $v_{\tau+}$, p_+, ρ_+ has been determined, it will be possible, making use of relations (4), to gradually determine the value of hydrodynamical magnitudes at other adjacent points of the stream. Facility of further solution of the problem is caused by the first of the equations (4) containing only a derivative of v_r, and consequently, in the numerical solution of v_r with the help of this relation, it may be eliminated from the two other equations (4). The plotting of the surface curvature of sharp rupture is to be done in the usual way.

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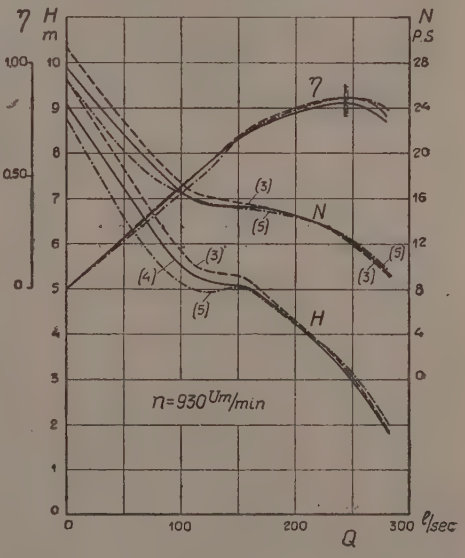
HYDRAULIK

DER EINFLUSS DER SCHAUFELZAHL EINER AXIALPUMPE AUF IHRE ARBEIT

Von P. A. WALTHER, korrespondierendem Mitglied der Akademie,
[und W. A. STEFANOWSKI

(Vorgelegt von S. A. Tschaplygin, Mitglied der Akademie, d. 3. III. 1938)

Wenn man eine axiale Pumpe entwirft, so setzt man gewöhnlich voraus, dass, erstens, die Strömung im Rad axial ist und, zweitens, dass diese Strömung bei dem normalen Betrieb mit gutem Wirkungsgrad sich von der Strömung der idealen Flüssigkeit nicht wesentlich unterscheidet. Diese Voraussetzungen berechtigen uns das Pumpenrad in eine Reihe von sogenannten elementaren Pumpen so zu zerschneiden, dass jede von diesen elementaren Pumpen einen bestimmten Halbmesser hat und *einem bestimmten hydraulischen Gitter entspricht.*



Um die Richtigkeit dieser Voraussetzungen zu prüfen, haben die Verfasser drei Pumpen geprüft. Diese Pumpen hatten gleiche Durchmesser des Rades und der Nabe, aber die eine von ihnen war mit drei, die andere mit vier und die dritte mit fünf Schaufeln versehen. Die Pumpenschaufeln waren so hergestellt, dass auf den Rechnungshalbmessern ihre zylindrischen Schnitte geometrisch ähnlichen Gittern entsprachen.

Wenn, folglich, die zu untersuchenden Voraussetzungen richtig sind, so müssen unsere Pumpen bei normalem Betrieb mit gutem Wirkungsgrad dieselben Ergebnisse zeigen. Nur ist zwischen den Wirkungsgraden selbst, welche bekanntlich von der Zähigkeit und folglich von den Reynoldschen Zahlen abhängen, ein kleiner Unterschied möglich. Bei Betriebszuständen, welche sich vom normalen Betrieb stark unterscheiden, kann der Unterschied zwischen der Arbeit der Pumpen sehr gross sein.

Die Prüfung hat diese Erwägungen bestätigt und folgende in der Figur dargestellten Resultate ergeben. Hier bedeuten (*3*), (*4*), (*5*) die Kurven, die sich auf Pumpen mit Schaufelzahlen 3, 4 und 5 beziehen.

Die Figur zeigt, dass die besten Wirkungsgrade der Pumpe, welche annähernd dem normalen Betrieb entsprechen, bei fast demselben Q erreicht werden, und dass die dabei ermittelten Förderhöhen H und die verbrauchten Motorleistungen N auch beinahe gleich sind. Der Unterschied zwischen den Pumpen besteht darin, dass bei drei Schaufeln der beste Wirkungsgrad $\eta_{max} = 0.84$ ist, bei vier Schaufeln $\eta_{max} = 0.82$ und, endlich, bei fünf Schaufeln $\eta_{max} = 0.845$.

Dieser Unterschied zwischen den Wirkungsgraden beweist natürlich, dass man ausser der gewöhnlichen Reynoldszahl

$$Re_a = \frac{nD^2}{\nu}, \tag{1}$$

noch eine andere Reynoldszahl

$$Re_i = \frac{l_m w_m}{\nu}, \tag{2}$$

betrachten muss. Hier sind:

l_m—die mittlere Schaufelbreite,

w_m—die mittlere relative Geschwindigkeit des Wassers innerhalb des Rades.

Die erste Reynoldsche Zahl bezieht sich auf die Grössen, welche die äusseren Pumpeneigenschaften charakterisieren. Deshalb soll sie die äussere Reynoldsche Zahl genannt werden. Die zweite bezieht sich auf die Eigenschaften der relativen Strömung innerhalb des Rades, und soll die innere Reynoldsche Zahl heissen.

Unser Versuch aus den erhaltenen Zahlen eine Abhängigkeit zwischen Re_i und den Eigenschaften des Rades bei dem Berechnungsbetrieb ($Q = 250$ l/sec) zu ermitteln, ergab die folgende Tabelle.

	Wassertemperatur (annähernd) °C	H, m	N, P. S.	η_{max}	Re_i
Rad *(3)*	15	3.05	12.10	0.84	$2.56 \cdot 10^6$
» *(4)*	10.5	2.98	12.15	0.82	$1.70 \cdot 10^6$
» *(5)*	18	3.13	12.30	0.845	$1.65 \cdot 10^6$

Folglich ist der Zusammenhang zwischen der inneren Reynoldschen Zahl und den hydraulischen Eigenschaften des Rades nicht eindeutig—zwei Räder (*4*) und (*5*), welche annähernd einem $Re_i \approx 1.70 \div 1.65 \cdot 10^6$ entsprechen, haben verschiedene Wirkungsgrade.

Da, übrigens, die Temperaturen während der Prüfung ungemessen blieben, so sind die in der Tabelle gegebenen Grössen Re_i ziemlich ungenau. Der mögliche Fehler kann bis zu 15% ausmachen und die Versuche müssen noch in dieser Beziehung verbessert werden.

Es wäre sehr interessant nachzuweisen, dass der Wirkungsgrad des Rades grösser wird, wenn die innere Reynoldsche Zahl zunimmt. Der gegenwärtigen Anschauung nach ist diese Vermutung sehr plausibel.

Die innere Reynoldsche Zahl nimmt zu, wenn man die Schaufelzahl verkleinert. Wenn also unsere Vermutung richtig wäre, so würde die Verkleinerung der Schaufelzahl den Wirkungsgrad des Rades vergrössern. Das könnte erklären, warum die Konstrukteure der letzten Jahrzehnte die Schaufelzahl der hydraulischen Maschinen immer zu verkleinern suchen.

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PHYSICS

THE FORM OF THE β-SPECTRUM OF RaE IN THE VICINITY OF THE UPPER LIMIT AND THE MASS OF THE NEUTRINO

By A. ALICHANIAN, A. ALICHANOV and B. DŽELEPOV

(Communicated by A. F. Joffe, Member of the Academy, 16. III. 1938)

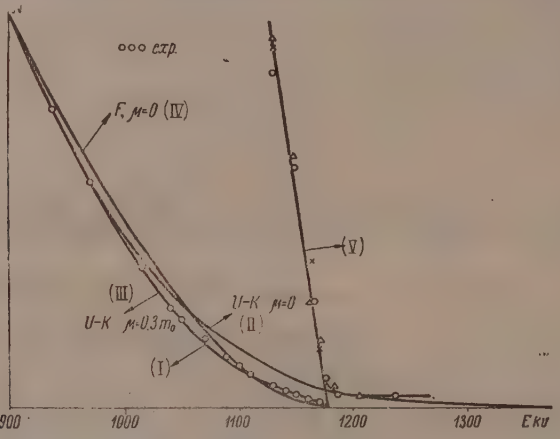
The spectrum of β-particles emitted by RaE has often been studied and it is now established(1) that over a wide range the form of the spectrum is expressed satisfactorily by the formula Uhlenbeck-Konopinsky. In the immediate neighbourhood of the upper limit, however, the spectrum has been studied in much less detail. Yet the results of Lyman's(2) measurements and those described in this note have proved the form of the spectrum in this region to depart considerably from that predicted by theory. The study of the energy distribution of the electrons in the neighbourhood of the spectral limit offers considerable difficulties. The first among them is the low intensity of the radiation in this part of the spectrum. The choice of apparatus and the sensitivity of the method of registering the electrons are therefore of prime importance. In the experiments described below the method of electron focusing in a uniform magnetic field has been used. The electrons were registered after the method of coincidence discharges in Geiger-Müller counters. It was necessary to create conditions which would not permit electrons scattered from walls of the apparatus and from diaphragms to enter the counters. This was attained by separating out a narrow beam of electrons ($\varphi=10°$) and by a special system of traps.

By placing the first counter with its narrow slit at the focus it was possible to eliminate almost absolutely the entry of scattered electrons. The source was a thin nickel strip on which 0.3—1 mC of RaE were deposited. The spectrum was studied in detail from $E=1030$ kV to $E_{gr}=1180$ kV. The results are represented in the figure from which it may be seen that the energy intervals were very narrow. The statistical accuracy with respect to individual points of the curve is satisfactory, the random errors being mainly due to measurements of the magnetic field. A conspicuous feature in the figure is the sharp change in the curve at an energy of about 1120 kV after which the number of electrons decrease practically linearly. This part of the curve is for clearness' sake reproduced on a larger scale in the same figure (V). The results of repeated experiments are indicated by different signs. The insignificant number of electrons observed at $E>E_{gr}$ are undoubtedly due to scattered electron and indicate that the experimental conditions were satisfactory.

The spectra computed according to the formula:

$$p \approx w\,(w^2-1)^{1/2}\,(w_0-w)^3\left[(w_0-w)^2-\left(\frac{\mu}{m_0}\right)^2\right]^{1/2} f(z,\,w),$$

which follows from the theory of Konopinsky and Uhlenbeck, are given in the figure. Curve *II* refers to the case $\mu=0$ and $w_0=3.69\ m_0c^2$, and Curve *III* to the case $\mu=0.3\ m_0$ and $w_0=3.69\ m_0c^2$. It is clear that Curve *III* agrees excel-



lently with the experimental results. It may be noted that the remainder of the experimental spectrum also fits the Uhlenbeck-Konopinsky theory satisfactorily when $\mu=0.3\ m_0$. The Curve *IV* has been computed according to Fermi's formula, using the experimental value of the spectral limit $w_{0exp}=3.31\ m_0c^2$, whilst μ was taken equal to zero. This curve does poorly agree with that found experimentally. The assumption that $\mu\neq 0$ does not lead to agreement between this theory and experiment for any value of μ.

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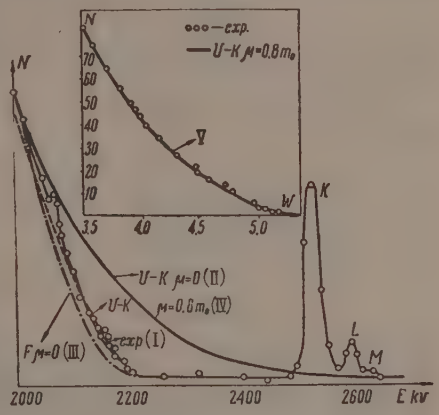
PHYSICS

THE FORM OF β-SPECTRUM OF ThC IN THE VICINITY OF THE LIMIT AND THE MASS OF THE NEUTRINO

By A. ALICHANIAN and S. NIKITIN

(Communicated by A. F. Joffe, Member of the Academy, 16. III. 1938)

Until recently it has always been regarded as an established fact that the energy spectrum of the β-particles emitted by any radioactive element approaches the spectral limit horizontally. Henderson(1), for example, found that the number of β-particles emitted by ThC and ThC'' fell of at the limit in such a way from which he concluded that the mass of the neutrino was nearly zero. Other investigators(3) have drawn the same conclusion from studies of the β-spectrum of RaE.



In contrast to these results A. Alichanian, A. Alichanov and B. Dželepov(2) found that the intensity curve of the electrons emitted by RaE cuts axis of abscissae at a certain angle. In particular it was found that the decrease between $E=1120$ kV and $E_{gr}=1180$ kV is practically linear. The authors were thus led to the conclusion that the mass of the neutrino is finite, since the tangent at $E=E_{gr}$ is not parallel to the axis.

It was therefore of importance to investigate whether the shape of the RaE spectrum is typical for all radioactive elements. The end of the spectrum of ThC was accordingly studied by the magnetic focusing method with registration by means of coincidences. The apparatus was the same as that used for RaE. A narrow beam of electrons with an angle of spread $\varphi=10°$ was focused on the slit of the first counter ($d\approx 0.8$ mm). A considerable number of diaphragms and traps prevented scattered electrons entering the counting system. An active deposit of Th (B+C+C'') (1—1.5 mC) was deposited on an aluminium strip 0.3 mm wide and 5 mm long.

The end of the spectrum of ThC is represented in the figure (Curve *I*). There is a distinct break in the curve at $E=2130$ kV, similar to that observed in the spectrum of RaE, after which the curve falls off practically linearly to the very limit. The small number of coincidences observed for $H > H_{gr}$ are

due partly to Compton electrons liberated from the walls of the vessel by γ-rays, and partly to scattered electrons and random coincidences. It is clear, however, that this background does not depend on the strength of the magnetic field. The remainder of the ThC spectrum down to $E=1275$ kV is given in the same figure (Curve *V*). By constructing the Konopinsky-Uhlenbeck plot from this spectrum (assuming the mass of the neutrino $\mu=0$) we found the extrapolated limit $w_0=6.18\ m_0c^2$ with the help of which the *U—K* spectrum for $\mu=0$ was constructed (Curve *II*). Curve *III* was computed according to Fermi's theory with $\mu=0$ using the experimentally determined spectral limit for the total energy of disintegration. Finally Curve *IV* has been computed according to the Konopinsky-Uhlenbeck theory for $\mu=0.8\,m_0$ from the formula

$$p \approx w\,(w^2-1)^{1/2}(w_0-w)^3\left[(w_0-w)^2-\left(\frac{\mu}{m_0}\right)^2\right]^{1/2} f(z, w),$$

where

$$w_0 = w_{0\,\mathrm{exp}} + 0.8\,m_0c^2.$$

It is clear that none of the theoretical curves cited, with the exception of Curve *IV*, agrees with the experimental spectrum. The satisfactory agreement between the latter and the Konopinsky-Uhlenbeck curve for $\mu=0.8\,m_0$ was not confined to the immediate vicinity of the limit but extended to the rest of the spectrum. A similar analysis in the case of the RaE spectrum showed that agreement between the theoretical and experimental curves is obtained when $\mu=0.3\ m_0$.

The following conclusions may be drawn from these results. The existing theory of Konopinsky and Uhlenbeck describes the experimental results correctly when $\mu\neq 0$; μ, however, is not a constant for all elements and varies with w_0.

It was found that the most suitable value of μ could be deduced from the equation:

$$w_{0\,\mathrm{extr}} - w_{0\,\mathrm{exp}} = \mu c^2,$$

which gives 0.8 m_0 for ThC and 0.3 m_0 for RaE. The fact, however, that μ varies from element to element means that no simple significance can be attached to this parameter.

Preliminary measurements of the end of the spectrum of RaC showed that in this case also the shape is similar to that in the vicinity of the spectral limit of RaE and ThC.

The three maxima observed in the figure at $E_1=2526$ kV, $E_2=2588$ kV and $E_3=2610$ kV are due to internal conversion of γ-line of ThC'' ($h\nu=2614$ kV) in the *K*, *L* and *M* levels of ThD. The sharpness of the lines bears witness to the high resolving power of the apparatus and the absence of disturbing factors. The relative intensities of the lines 5 : 1 : 0.2 give the relative probabilities of conversion of the γ-line $h\nu=2614$ kV in the *K*, *L* and *M* levels. The small maximum at $E=2070$ kV corresponds to the internal conversion of the γ-line $h\nu=2160$ kV found by Alichanov and Dželepov in the positron spectrum of Th (C+C''). The present measurements yield a value of $\alpha=1.77\cdot10^{-3}$ for the coefficient of internal conversion of the γ-line $h\nu=2614$ kV whereas according to Taylor and Mott[4] the theoretical value is $\alpha=1.5\cdot10^{-3}$.

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PHYSICS

ON THE ANGLES BETWEEN THE COMPONENTS OF A PAIR

By A. ALICHANIAN, B. DŽELEPOV and P. SPIVAK

(Communicated by A. F. Joffe, Member of the Academy, 25. III. 1938)

The phenomenon of pair-creation by γ-rays has been the subject of a number of investigations, in most of which the energy spectrum of the particles was measured and the conversion coefficient of the γ-rays determined. The problem of the angle between the paths of the pairs has received practically no attention. In order to study this aspect of the

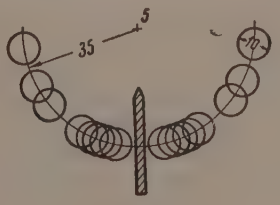

Fig. 1.

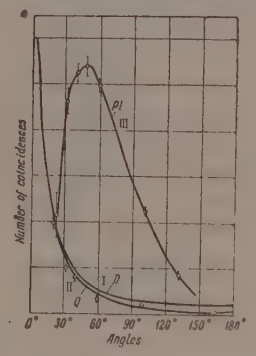

Fig. 2.

phenomenon we have investigated the probability of flight along different direction for pairs created by γ-rays (Po+Be).

About 20 mC of Po carefully freed from RaE were deposited electrolytically on a thin foil (~30 μ) of Al—Be alloy (65% Be+35% Al). The pairs were registered by the method of coincidences in two small Geiger-Müller counters ($l=30$ mm, $d=10$ mm).

The first experiments showed that the source described above emits an appreciable number of pairs which must be attributed to the internal conversion of the γ-rays radiated by the source. Owing to the high resolving power of the counters with respect to coincidences, the number of random coincidences was negligible, in spite of the fact that the pairs and the electrons from traces of RaE gave up to 200 discharges per minute, and the background could be attributed to cosmic rays.

The pairs were measured at six different angles (Fig. 1); the corresponding results are represented by circles in Fig. 2. Since, according to Bothe (1), the source (Po+Be) emits two softer lines 4.5 and 2.5 MeV in addition to the hard γ-rays at about 7 MeV, the counters were surrounded by aluminium cylinders ca 0.5 mm thick in order to eliminate the effect of the soft rays.

The absorption of the pair components in the Al cylinders is taken into account by introducing a suitable correction. The significance of the results obtained is represented in Fig. 2 by vertical lines.

It is possible to calculate the angular distribution of pairs created by internal conversion from the theory developed by Uhlenbeck and Rose(2). The theoretical curves for dipole and quadrupole transitions are given in Fig. 2, where the scale has been chosen so that the first experimental point lay on both curves. The general agreement with the theoretical curves is fully satisfactory. Although no final decision as to the type of radiation can be made on the basis of these results, yet we may state that the experimental points lie more closely to the curve for quadrupole transitions.

The angles between pairs created by γ-rays (Po and Be) in lead were investigated under similar conditions. The source in this case was surrounded by a lead foil (54 μ). The experimental points in Fig. 2 correspond to the increase in the number of pairs as compared to the first series of experiments and a small correction has been introduced for the absorption of the components of the pair in lead by internal conversion. A strong maximum occurs in the curve in the region 45—50°. Unfortunately the theory of pair creation by γ-rays gives such a complicated formula for the angular distribution that a comparison with the data experimentally obtained presents considerable difficulties.

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PHYSICS

THE STATE OF THE NICKEL ATOM IN THE GAMMA-PHASE OF THE Ni—Zn SYSTEM

By J. DORFMAN and S. SIDOROF

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The application of the well-known Hume-Rothery rule to gamma-phases consisting of Zn (or Cd) alloyed with an element of the VIII group (Fe, Co, Ni, Pd, Pt, etc.) leads usually to a very curious conclusion.

It seems, namely, that while Zn (or Cd) behaves normally, giving off its 2 valence electrons, an element of the VIII group gives no electrons at all to the metallic body of the gamma-phase, remaining there as a neutral atom.

The purpose of this work was to check this unusual supposition by studying the magnetic properties of those gamma-phases. We thought to measure the magnetic moment of the Ni atom which would permit us to determine the state of the Ni atom in the alloy.

We investigated the alloys Ni—Zn by means of a special improved horizontal magnetic balance of the Weiss-type. The details of the apparatus and the experimental procedure will be published elsewhere. The results are represented in the table.

№ of the sample	1	2	3	4	5
Ni content in atomic %	20.25	19.50	18.3	17.0	16.1
Mass susceptibility at room t° . . .	— 2.02	— 2.61	— 3.62	— 5.45	— 6.85

The samples have been controlled by X-rays and proved to represent allmost pure gamma-phases. Only in samples with a Ni content lower than 16.1% besides the gamma-phase other phases appear too.

The samples were prepared from Mond (Ni) and Kahlbaum (Zn) and were tempered for several days. No influence of the intensity of the magnetic field on the χ values has been observed, this result proving the absence of grains of free metallic Ni in the alloy. The susceptibility of the samples 1 and 2 is slightly reduced by heating, while that of other samples remains independent of temperature. The gamma-phase in the Ni—Zn possesses a great diamagnetism characteristic of normal gamma-phases. Thus, we are led to the conclusion that the magnetic moment of the Ni atom in this given gamma-phase is equal to zero.

But this value of the magnetic moment can be obtained with a neutral Ni atom only, possessing 28 electrons just sufficient to fill out eventually all the corresponding electronic levels.

It follows that the Hume-Rothery rule is in excellent agreement with our results, the Ni atom being in the $^1S_0$ state in the gamma-phase alloy.

The investigation will be repeated on analogous alloys containing Fe or Co atoms.

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PHYSICS

THE NEW «POSITIVE» BARRIER PLANE PHOTOELECTRIC EFFECT AND THE NEW BARRIER PLANE PHOTOCELL

By B. KOLOMIEZ

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The barrier plane photoelectric effect, discovered towards the end of the last century, was subjected to a detailed investigation only in the last ten years when the reproducible technique for obtaining those photocells with selenium and cupreous oxide has been worked out.

The action of those photocells is based on the fact that under the influence of light the electrons pass from the semi-conductor into the metal through a thin barrier plane with a high resistance. In consequence of what the metal of the electrode is charged negatively and the semi-conductor is charged positively. In the cupreous oxide photocells in case of the front wall effect the upper illuminated electrode is charged negatively, and the electrode having a coating of the semi-conductor positively. In case of the back wall effect the barrier plane is situated between the cupreous oxide and the copper. When the electrode is illuminated, the electrones pass into the copper and the upper electrode is charged positively. The metal adjoining the barrier plane in all known cases is charged negatively. Technically fit photocells were up to now manufactured with cupreous oxide and selenium. The coefficient of utilisation of the light energy is in those cells considerably less than 1%. Therefore it is of great interest to obtain photocells with other semiconductors having higher indices. If we consider the internal photoelectric effect as the value that can characterize the fitness of a given material for manufacturing barrier plane photocells, the thallium sulphide would be the best semi-conductor for this purpose.

The thallium sulphide is used since 1928 in resistance photocells and many experimental investigations of it have been carried out. One of the last works is that by B. Hochberg and M. Sominsky, in which it is shown that if subjected to appropriate treatment the thallium sulphide can have both the electron character and the «hole» character of conductivity. This curcumstance, as proved by our experiments, is very important.

If the thallium sulphide possesses the «hole» conductivity, it has the usual barrier plane photoelectric effect corresponding to the passage of electrons from the semi-conductor into the metal. In this case there is no difference from cupreous oxide and selenium which, as well known, also have the «hole» conductivity.

But if the thallium sulphide is in state of «electron» conductivity, it also has the barrier plane photoelectric effect but with the inverse sign. In this

case the electrons pass from the metal into the semi-conductor, replacing the free energetic levels, and therefore the metal adjoining the barrier plane is charged positively and the semi-conductor negatively. It was found that such photocells with the photoelectric effect of positive sign possess very high technical properties, as is shown by preliminary data on the new «thallium—sulphide» photocells having «the positive» photoelectric effect.

1. The sensitivity of the thallium sulphide photocells at illumination by glow lamps is equal to 5000—6000 μA per lumen, i. e. is 10 times higher than that of the selenium photocells under the same conditions of illumination.

2. The electromotive force attains 0.3 V.

3. The dependence of the short circuit current and of the electromotive force of the thallium sulphide photocell on the illumination is the same as that of the selenium and cupreous oxide photocells.

4. The character of the photocurrent on different resistances in dependence on illumination does not differ from the already known photocells either.

5. The spectral sensitivity of the thallium sulphide barrier plane photocells does nor differ from the sensitivity of the «tallofides».

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PHYSICS

EXCITATION OF METASTABLE ATOMS IN A GAS DISCHARGE

By V. FABRIKANT

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In the two preceding works([1],[2]) two basic types of collisions between electrons and atoms in a gas discharge have been discussed.

In the present paper an attempt is made to build up the principles of a quantitative theory of the excitation of atoms in a gas discharge.

In carrying out this task we shall to a certain extent make use of conceptions evolved in works on the optical excitation of atoms([3]).

1. In a gas discharge the concentration of excited atoms is determined by the following factors: 1) excitation by collision of the first kind between electrons and atoms; 2) diffusion of atoms towards the walls of the discharge tube; 3) destruction under collision of the second kind with electrons and atoms.

In the case of a stationary discharge representing an infinite cylinder, the concentration of excited atoms should satisfy the differential equation:

$$D_a\left(\frac{\partial^2 n_a}{\partial r^2}+\frac{1}{r}\frac{\partial n_a}{\partial r}\right)+\alpha_a n q_e=\beta_a n_a q_e+\gamma_a n_a, \qquad (1)$$

where n, n_a are concentrations of normal and excited atoms; q_e, concentration of electrons at a given distance from the axis referred to the concentration on the axis; D_a, the coefficient of diffusion of excited atoms; α_a, the probability of collisions of the first kind between electrons and atoms on the axis; β_a, the probability of collisions of the second kind between excited atoms and electrons on the axis; γ_a, the probability of collisions of the second kind with atoms. In full analogy with optical probabilities we understand here under probabilities the number of collisions effected by an atom within a definite unit of time.

We shall seek for the solution of equation (1) under the following boundary conditions:

$$\left.\frac{\partial n_a}{\partial r}\right|_{r=0}=0; \quad \left.n_a\right|_{r=a}=0 \qquad (2)$$

where a is the radius of the tube.

W. de Groot examined in detail the range of application of boundary conditions corresponding to the type in question and for analogous problems([4]).

As usual we shall assume that q_e is Bessel's function of zero order([3]).

2. Let us consider two limiting cases: 1) $\gamma_a \gg \beta_a$; 2) $\beta_a \gg \gamma_a$. The first case corresponds to the destruction of metastable atoms by various admix-

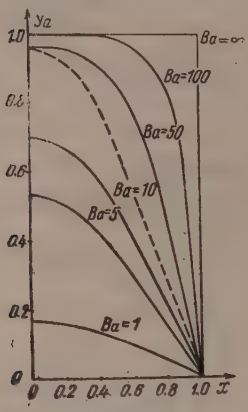
tures; the second case corresponds to discharges with high current densities, such as are to be found in pure vapours and gases.

In the first case:

$$n_a(r) = n_a(0)\, I_0\left(\mu_1 \frac{r}{a}\right); \quad n_a(0) = \frac{\alpha_a n}{\frac{\mu_1^2 D_a}{a^2} + \gamma_a}. \tag{3}$$

In other words, the excited atoms are distributed over the cross-sections of discharge according to Bessel's function of zero order. The maximum concentration is as usual determined as a product of the number of exciting collisions and of mean life.

The second limiting case is considerably more complicated. If in the discharge Maxwell's distribution according to velocities is adhered to, a fixed relation between α_a and β_x will be found to exist and equation (1) may be given the following universal form:



$$\frac{1}{x}\frac{d}{dx}\left(x\frac{dy_a}{dx}\right) + B_a q_e = B_a q_e y_a, \tag{4}$$

where y_a is the «reduced» concentration equalling the ratio of the concentration of excited atoms available under the given conditions to concentration setting in with Boltzmann distribution which corresponds to the same electronic temperature; $x = \frac{r}{a}$; $B_a = \frac{\beta_a a^2}{D_a}$ is proportional to the ratio of the probability of collisions of the second kind to the probability of diffusion (the proportionality coefficient is μ_1^2). The introduction of the notation «reduced» concentration affords a great advantage, this magnitude changing but slightly with electronic temperature.

Fig. 1 represents the solution of equation (4) with different values of the parameter B_a.

With small B_a (< 10) the distribution curve differs but slightly from Bessel's function of zero order. With increase of B_a there is also to be found a definite enhancement of the part taken by collisions of the second kind in the destruction of the excited atoms and, owing to saturation, the distribution curve will be found growing more convex. With $B_a = 50$ a strong deviation from Bessel's function is observable (plotted in a dotted line to facilitate comparison). With $B_a = 100$ it already becomes possible to resort to an asymptotic solution of equation (4) having the following appearance:

$$y_a = 1 - \frac{I_0(\sqrt{B_a}x)}{I_0(\sqrt{B_a})}, \tag{5}$$

where I_0 is Bessel's function from a purely imaginary argument(7). From (5) it follows that with $B_a \to \infty$, $y_a \to 1$ throughout the section of the discharge and the distribution curve terminally passes into a horizontal straight line. Thus, with $B_a \to \infty$ temperature equilibrium sets in, and the concentration of excited atoms is determined by Boltzmann's formula (8). «Reduced» concentration is in this sense analogous to emitting power in optics.

For a number of questions it will be sufficient to know the mean concentration of excited atoms in the discharge. In the first of the cases examined this magnitude is determined in a most obvious manner; in the second case (making allowances for the part taken by separate layers of the discharge) it may be possible to show that when $B_a < 50$:

$$\bar{y}_a \approx 0.44 \frac{B_a}{\mu_1^2 + 0.66 B_a}. \tag{6}$$

Thus, $\bar{y}_a$ is a function of parameter B_a only.

3. Let us now consider how discharge conditions affect B_a.

For this purpose it will be necessary to write β_a in explicit form, which can be done by making use of the results outlined in a paper formerly published (2).

$$\beta_a = 7 \cdot 10^7 \cdot Q_{21}(0)\, n_e(0) \sqrt{T_e} f(T_e), \qquad (7)$$

where $Q_{21}(0)$ is the effective section of excited atoms for collisions with electrons of the second kind; $n_e(0)$ is the concentration of electrons on the discharge axis; T_e, electronic temperature (11 600° K = 1 V); $f(T_e)$, a factor expressing the dependence of the probability of collisions of the second kind upon electron velocity.

Consequently, β_a may be roughly considered as proportional to current density, while B_a will be proportional to current (current density $\times a^2$). It being given that D_a is inversely proportional to gas pressure, B_a will ultimately be proportional to the product of current times pressure (this relation holds true only approximately since pressure affects T_e). From (6) the inference can be made that with equal pressures and small B_a «reduced» concentrations of excited atoms in different discharge tubes will be directly proportional to the respective currents.

With enhanced pressure «reduced» concentrations will usually increase too.

4. Let us quote some instances of the concrete application of the theory advanced above.

The best check of the theory lies in a calculation of absolute concentrations of excited atoms. In one of the earlier works of our laboratory a definition was made of absolute concentrations of excited atoms in low pressure mercure discharges(9).

Since the measurements were carried out in a narrow tube ($a = 1.6$ cm), there is reason to compare theory with experimental evidence only for the highest of the pressures investigated ($7.5 \cdot 10^{-3}$ mm Hg). For concentrations at level 6^3P_0 calculation gives $5 \cdot 10^{11} \frac{\text{atm}}{\text{cm}^3}$, while the experimental value stands at $4.9 \cdot 10^{11} \frac{\text{atm}}{\text{cm}^3}$; for level 6^3P_2 calculation gives $1.5 \cdot 10^{12} \frac{\text{atm}}{\text{cm}^3}$, whereas the experimental value is $1.4 \cdot 10^{12} \frac{\text{atm}}{\text{cm}^3}$. The magnitudes of the elementary constants were taken from the same sources as in our foregoing works.

5. On the ground of the aforesaid considerations, it may be possible to show that in such discharges as are used in powerful mercury rectifiers, the concentrations of metastable atoms should approach the Boltzmann concentrations. For instance, with $i = 800$ A, $a = 15$ cm and $p = 7.5 \cdot 10^{-3}$ mm Hg, the parameter of $B_a \approx 200$. Under such conditions the excited atoms ought to constitute about 10% of the total amount of atoms in the discharge. This explains the great significance of cumulative processes in such discharges.

6. Finally, let us dwell upon one more question. In the well-known work of Meissner and Graffunder(10), devoted to a determination of the life period of metastable atoms of neon, the assumption is made that at the initial moment (i. e. in a stationary discharge) the excited atoms are uniformly distributed along the discharge section. These initial conditions complicate the solution of the problem and compel the authors to perform a number of rather cumbersome calculations in order to show that under these conditions, beginning from a certain time, an exponential abatement of concentration practically takes place. The relationships stated above show that under the conditions of Meissner and Graffunder's experiments at the initial moment atoms are distributed over the section of the discharge not in

an uniform way, but according to Bessel's function. Then, as is well known, the concentration is to abate along the exponent and all complications connected with wrong initial conditions assumed in the mentioned work fall off of themselves.

7. The considerations stated above show that Schottky's theory of a positive column[4] should be supplemented by proper evaluation of the processes of cumulative ionization. These processes being taken into account, the differential equation for concentration of electrons will be analogous to equation (1), but differing from it by the sign of the additional term. It is quite natural that cumulative ionization will be found to be stronger on the axis of the discharge than at the walls, which ought correspondingly to affect the distribution of electrons over the discharge section.

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PHYSICS

EXCITATION OF RADIATING ATOMS IN A GAS DISCHARGE

By V. FABRIKANT

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1. Some years ago W. de Groot and Druyvesteyn attempted to calculate theoretically the concentrations of excited radiating atoms in a sodium discharge(1). This attempt ended, however, in failure, since the course taken by the authors met with excessive mathematical difficulties(2).

In the present work we shall resort to another way based on an excellent analogy between the diffusion of quanta and atoms suggested by Compton(3). Though this course is not altogether a strict one, it has the advantage of simplicity and affords the possibility to make use of the results obtained in the preceding work(4).

The inexactitude of the analogy drawn by Compton was pointed out by Milne in his fundamental work(5) which has since served as the main basis for the quenching theory of excited atoms under optical excitation(6). Milne advanced a more strict theory of diffusion of prisoned radiation in gas. However, it may be shown that this theory under certain conditions (which are practically the most important) will be equivalent to the theory based on the analogy suggested by Compton.

2. Milne examined the diffusion of radiation in an infinite plane layer after the excitation was eliminated and obtained the following differential equation:

$$\frac{\partial^2}{\partial x^2}\left(n_r + \tau \frac{\partial n_r}{\partial t}\right) = 4k^2\tau \frac{\partial n_r}{\partial t}, \qquad (1)$$

where n_r is the concentration of excited radiating atoms; k, the coefficient of gas absorption; τ, the life time of an excited radiating atom.

In deducing his equation (1) Milne assumed that only normal atoms are capable of absorbing radiation. Now, let us in the very first place observe that Milne obtained equation (1) while making use of Schuster-Schwarzschild's approximation(7) which is virtually equivalent to the assumption that the intensity of a radiation flux in a given medium is independent of the direction of that flux relatively to the axis of the x. In the case of a plane diffusion problem it would be more natural to consider this intensity to be proportional to the cosine of the angle between the direction of the flux and the axis of the x (along which a gradient of radiation density and a concentration of radiating atoms are observable). Then it may be easily shown that equation (1) assumes the form:

$$\frac{\partial^2}{\partial x^2}\left(n_r + \tau \frac{\partial n_r}{\partial t}\right) = 3k^2\tau \frac{\partial n_r}{\partial t}. \qquad (2)$$

Equation (2) differs from the ordinary equation of diffusion by the presence of term $\frac{\partial^3 n_r}{\partial x^2 \partial t}$ in its left-hand side.

Let us now apply equation (2) to the excitation of radiation in a discharge. Since in Milne's theory $\frac{\partial n_r}{\partial t}$ accounts for variations in the number of excited atoms occasioned by optical processes only, it is clear that in the case of the existence of other excitation processes $\frac{\partial n_r}{\partial t}$ should be replaced by $\frac{\partial n_r}{\partial t} - R$, where R is velocity of formation of excited atoms, produced at the expense of non-optical processes. But should we examine a stationary discharge, $\frac{\partial n_r}{\partial t}$ will be equal to zero and equation (2) will have the following appearance:

$$\frac{\partial^2}{\partial x^2}(n_r - \tau R) = -3k^2 \tau R. \tag{3}$$

Just as in equation (1) of the preceding paper, we shall represent R as the difference between the number of collisions of the first and the second kind:

$$R = \alpha_r q_e n - \beta_r q_e n_r. \tag{4}$$

However, since we are dealing here with a plane problem, we shall express q_e as follows $\left[x \text{ changing from } -\frac{l}{2} \text{ to } +\frac{l}{2} \right]$:

$$q_e = \cos \frac{\pi}{l} x. \tag{5}$$

Making use of (4) and (5) we can give equation (3) the following form:

$$(1 + \tau \beta_r q_e) \frac{\partial^2 n_r}{\partial x^2} = -\left(3k^2 \tau + \frac{\pi^2}{l^2} \tau\right)(\alpha_r q_e n - \beta_r q_e n_r); \tag{6}$$

when

$$\tau \beta_r q_e \ll 1 \tag{7}$$

equation (6) passes into:

$$D_r \frac{\partial^2 n_r}{\partial x^2} = -(\alpha_r q_e n - \beta_r q_e n_r), \tag{8}$$

where

$$D_r = \frac{1}{\left(3k^2 \tau + \frac{\pi^2}{l^2} \tau\right)}. \tag{9}$$

3. Equation (8) is exactly of the same type as equation (1) which has been obtained for metastable atoms. It will be shown in the subsequent that, under the conditions of discharge in hand, inequality (7) is almost always fulfilled. The physical meaning of this inequality is very simple—the number of second kind collisions suffered by an excited atom during the whole of its life should be less than unity. Besides this, if we confine ourselves to a consideration of a discharge with great optical densities, where

$$kl \gg \frac{\pi}{\sqrt{3}} \tag{10}$$

then

$$D_r = \frac{1}{3k^2 \tau}. \tag{11}$$

Relation (11) is obtainable from the above mentioned analogy for the diffusion of quanta and atoms[3].

Thus, it is only with low optical densities that the difference between Milne's theory and the usual diffusion theory becomes manifest. This dif-

ference will be seen to lie in the fact that with $k \to 0$ the diffusion coefficient remains finite $\left(\frac{l_2}{\tau^2}\frac{1}{\tau}\right)$. With higher densities, radiation diffusion is equivalent to the diffusion of radiating atoms with a diffusion coefficient determined by formula (11).

From the above stated it follows that the excitation of radiating atoms in a cylindrical discharge column with high optical densities can be described by an equation fully identical with equation (1) of the preceding work; only D_r will in this case be determined by formula (11).

On this account, all the conclusions arrived at in the former work will also hold true for radiating atoms. It should, however, be emphasized that the problem of excitation of radiating atoms is virtually by far less accurately solved than for metastable atoms. The reason lies in the fact that in the case of radiation diffusion a highly important part is played by the very form of the lines—a factor whose influence can be accounted for only with difficulty. Accordingly we shall be compelled, as is usual for the case in hand, to restrict ourselves to an approximate evaluation of the part taken by the form of lines, which will naturally not be without affecting the accuracy of the results obtained.

4. As is well known, due account taken of the form of the lines leads to a non-exponential law of weakening of the radiation bundle as it passes through a gas([8]). Deviations from the exponential law correspond to a continuous decline of the absorption coefficient. Milne's theory, as well as the ordinary diffusion theory, provide for the absorption coefficient being constant. Yet, even in the case of the simplest form of the lines, the absorption coefficient varies in a highly complicated way which, in the deduction of the basic differential equation, makes the proper evaluation of this variation highly difficult. Therefore, it is usual to confine the calculation of the differential equation with constant absorption coefficient to the introduction into the ready solution of the ferm of effective optical density for the whole layer, this density being so calculated as to take due account of the form of the lines([9]). This course, naturally, fails to be a thoroughly strict one but, as is borne out by comparison with experimental evidence, it actually yields sufficiently accurate results([10]). With high optical densities the coefficient of radiation diffusion calculated by Kenty([11]) should be made use of. An additional small inaccuracy is, however, brought about by the fact, that Kenty determined the diffusion coefficient for a plane layer, instead of for a cylinder, but we may neglect this. Then it can be shown that the effective life time τ_r will be determined by the following approximate relation:

$$\tau_r = 3k_0 a\tau, \tag{12}$$

where k_0 is the coefficient of absorption in the middle of the life. Since k_0 is inversely proportional to τ, τ_r is virtually independent of τ!

5. Acting in exactly the same way as in the preceding work, we can introduce parameter B_r and then draw a number of conclusions on the influence of discharge conditions upon the excitation of radiating atoms:

a) From the numerical values of the coefficients entering the respective formulae it follows that with similar discharge conditions $B_r < B_a$.

Thus, the distribution of radiating atoms differs more markedly from the equilibrium value than the distribution of metastable atoms.

b) Reasoning in the same manner as in the preceding work, we obtain for B_r the following approximate relation:

$$B_r \approx \frac{ip}{a}. \tag{13}$$

This relation shows that with weak quenching the «reduced» concentration of radiating atoms is directly proportional to the ratio of current to tube radius (pressure being constant). This latter circumstance distinguishes the radiating from metastable atoms.

c) The quenching of radiating atoms by collisions of the second kind becomes observable only with strong «prisoning» of the radiation, when $\tau_r \gg \tau$. The «prisoning» of radiation is one of the main factors determining the quenching of radiating atoms in a discharge.

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PHYSICS

ON THE EFFECT OF MAGNETIC FIELD UPON MERCURY DISCHARGE RADIATION

By V. FABRIKANT and G. ROKHLIN

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1. The present work is a first attempt at making a quantitative investigation of the effect of a magnetic field upon the intensity of radiation of a gas discharge.

The literature includes a number of works devoted to the question of the influence of a magnetic field upon a gas discharge, but all these works deal pre-eminently with the electrical characteristics of the discharge(1). There exists, in fact, only a single exception consisting of a work in which a very special type of discharge was investigated and the magnetic field was applied in obtaining intensive spark spectra(2). However, in that work merely methodical problems were considered.

At the same time the question of the influence of a magnetic field upon the luminescence of an electric discharge is for quite a number of reasons a subject of outstanding interest both from the theoretical and the practical points of view (the problem of northern lights, the provision of new sources of light, etc.).

2. For our work we made use of a low-pressure mercure lamp connected to a d. c. supply. The lamp possessed a liquid cathode and vapour pressure in it was kept up with the help of water cooling. The lamp was 30 mm in diameter and 240 mm in length. For obtaining the highest freedom from impurities the lamp was all the time connected with a vacuum set.

Studies were made of the influence of a magnetic field on a positive discharge column. The magnetic field was set up by means of two solenoids mounted upon the middle portion of the lamp. The length of the solenoids was 55 mm with an inside diameter of 60 mm and an outside diameter of 130 mm.

The solenoids were set at different distances from one another. Measurements of radiation intensities were accomplished in the interval between the solenoids.

Intensity measurements were made with the help of a König-Martens spectrophotometer. For measuring the intensity of resonance lines, of 1850 Å and 2537 Å respectively, a luminescent probe was inserted into the lamp. The probe was located at the inner wall of the lamp. In measuring the visible lines an opal glass was placed between the lamp and the spectrophotometer. In measurements of the brightness of the probe, its image was focused on the slit of the spectrophotometer. Measurements were effected with

two different configurations of the field: first, in a longitudinal homogeneous field with solenoids switched in in series, and secondly, in a strongly non-homogeneous field produced by counterwise switching in of the solenoids. The influence of such a non-homogeneous field possessing axial symmetry was, in so far as we know, not studied hitherto, notwithstanding the fact that it gives the most appreciable effect (see below). It is clear that the radial components of the magnetic field play the principal part in this case. In Fig. 1 and 2 are to be found the averaged results of several series of measurements.

3. Measurements in the longitudinal field were effected with the distances between the solenoids equalling 25 mm which, with the given dimensions of the solenoids, secured sufficient homogenity of the field. In switching in the solenoids the discharge was counteracted.

In Fig. 1 are to be found the results of measurements at a temperature of 16° C and with a discharge current of 1.5 A. Current in solenoids and intensity of magnetic field are plotted on the abscissas, while on the axis of the ordinates are to be found the intensities of the lines 5770—91 Å and 5461 A and the brightness of the probe expressed in conventional

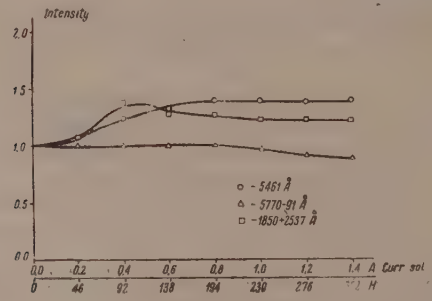

Fig. 1.

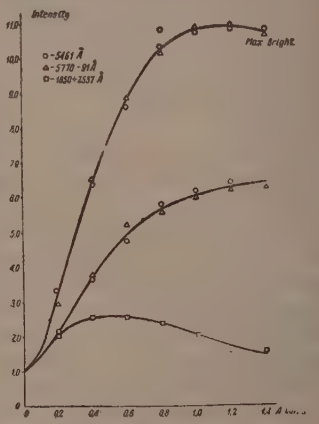

Fig. 2.

units. For greater ease of comparison, all three curves are made to coincide at the initial point.

Measurements in the non-homogeneous field were performed with different distances between the solenoids.

Fig. 2 represents the results obtained with the distance between the solenoids being 30 mm (16° C, 1.5 A).

The same values as in Fig. 1 are plotted along the axes of the coordinates.

With smaller distances between the solenoids the increase in intensity was practically the same. Increasing the temperature of the lamp cathode and increasing the amperage of the discharge current tended to lower the effect.

4. From the curves of Fig. 1 it follows that a longitudinal magnetic field has but small influence upon the general intensity of discharge radiation. The maximum increase in intensity comes to 40% of the initial magnitude.

Per contra, from the curves of Fig. 2 it follows that a non-homogeneous magnetic field very greatly affects the intensity of radiation, a great difference in the behaviour of the visible lines and resonance radiation (brigthness of the probe) being observable. The general intensity of lines, 5461 Å and 5770—91 Å, rises six or sevenfold, while the intensity of resonance radiation alters within a much smaller range, reaching its maximum value

with the current in the solenoids being 0.5 A. At the maximum, the intensity of resonance radiation is 2.5 times as high as the initial intensity, while with greater currents in the solenoids it is only 1.5 times as high. Thus, in a non-homogeneous magnetic field there is to be observed a sharply marked re-distribution of energy in the spectrum of a mercury lamp.

In such a field the area of the positive column lying between the solenoids assumes the aspect of an ellipsoidal luminous body.

Measurements of the course of maximum brightness made without an opal glass, with current switched in in the solenoids, have shown this brightness to grow yet faster than general intensity. The maximum brightnesses of lines 5461 Å and 5770—91 Å, with the same range of changes in solenoid current as above, rise about eleven or twelvefold.

It is highly important to note that the character of the effects observed failed to change in any way with reversal of current motion in solenoids and also with substitution in solenoids of alternating for direct current.

5. In the theoretical interpretation of the above stated effects particular importance belongs to the great length of electron free path as available under our experimental conditions [a few centimeters(3)]. The greater the electron free path, the stronger is the effect.

The function of a magnetic field consists, in the main, in causing distortions of electron paths and in increasing the number of collisions between electrons and atoms. Considering the matter from this standpoint we can easily understand the difference observed in the action of a longitudinal and non-homogeneous field. In a non-homogeneous field the outline of the luminous column is similar to the picture of the magnetic field; and the observed effects considered in this sense may be said to be analogous to the effects examined in Störmer's theory of northern lights(4).

The highest theoretical interest belongs to the observed differences in the course of the intensities of separate lines. With a longitudinal field, the observer is struck by the difference in the behaviour of visible lines 5461 Å and 5770—91 Å. Line 5770—91 Å, possessing a somewhat higher potential (8.8 V) than line 5461 Å (7.7 V), will even be slightly lowered with increase of field.

The intensity course of resonance radiation is to a certain extent similar to the course of the gradient observed under similar conditions by Spivak and Reichrudel(1). This similarity cannot be explained by mere chance, in so far as in the discharge conditions under investigation a considerable share of discharge power is expended on resonance radiation(5).

The difference in the behaviour of resonance and visible lines in the case of a non-homogeneous field may, obviously, be accounted for by secondary processes: cumulative excitation and quenching by second kind collisions with electrons.

The influence of a magnetic field is in this sense equivalent to an increase in discharge current density(6).

For providing a further univocal interpretation of the mechanism of excitation in a magnetic field, special experiments were set up, making use of tubes of a design already applied in our laboratory for absorption measurements(7). This design eliminates the rapid electrons flying from the side of the cathode and also permits to establish close control of pressure. Under these conditions the counteraction of a discharge in a longitudinal field will be investigated quantitatively.

6. The phenomenon found by us and consisting in a non-homogeneous magnetic field having a considerable influence upon the luminosity of a discharge may prove of high advantage for various practical applications. The total amount of light emitted by the mercury discharge is practically

concentrated in the lines 5461 Å and 5770—91 Å, so that the observed increase in the intensity of these lines is equivalent to a tantamount increase in brightness and light output of the lamp.

Changes in the brightness of the lamp (modulation) may prove useful of themselves, being serviceable in a number of cases, but what is particularly important is that every increase in brightness will be accompanied by a very considerable augmentation of the intensity of illumination produced by the lamp. Measurements have shown that with the application of a magnetic field the power consumed by the lamp increases no more than fifty per cent, i. e. is increasing several times slower compared to the intensity of illumination.

Consequently, the application of a non-homogeneous magnetic field may be considered as a new method for raising the light output of the gas discharge lamp.

One of the main reasons for such higher light output lies in the sharply marked redistribution of energy in the magnetic field available in the spectrum of a mercury lamp.

As regards the energy expended on maintaining a magnetic field, this may practically be left out of calculation, seeing that the solenoids may be switched in in series with the lamp as a necessary resistance or else they may be replaced by permanent magnets.

For the practical application of the results obtained it will, of course, be necessary to further improve the design of the lamp and to find such conditions as would correspond to the possible maximum of light output.

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PHYSICS

THE MAGNETIC ANISOTROPY OF FERROMAGNETIC CRYSTALS IN WEAK FIELDS (1)

By E. KONDORSKY

II. THEORY OF THE REVERSIBLE SUSCEPTIBILITY OF FERROMAGNETIC CRYSTALS

(Communicated by S. I. Wawilow, Member of the Academy, 17.III.1938)

The concept of reversible susceptibility was introduced into the theory of magnetism by Gans (2) who defined this magnitude and measured it for various materials as a function of the magnetization. On the basis of his investigations Gans expressed the supposition that the ratio $\frac{\chi_r}{\chi_0}$, where χ_0 is the initial susceptibility, is one and the same function of $\frac{I}{I_s}$ for all materials. Later (3) he deduced a formula for this function which is in good agreement with experiment for iron and nickel, but which proved to be inapplicable to cobalt (4). Measurements of the reversible susceptibility of crystals in different directions would be of theoretical significance, but have not yet been carried out owing to the difficulty of taking into account the demagnetizing factor. An attempt is made below to build up a theory of reversible susceptibility in weak fields (i. e. in the «region of inversion») from the standpoint of modern conceptions of the process of magnetization.

There are two processes which take place during magnetization: 1) the process of rotation of the resultant spins of domains (or regions of spontaneous magnetization) from the direction of easy magnetization into the direction of the field, and 2) the process of displacement of the boundaries between domains and the engulfment of some domains by others. In weak fields the second process plays the principal part. As has already been indicated (1), two types of boundary displacement should be distinguished in this second process: 1) displacement of the boundaries between domains with spins at an angle of 90° or, at least, at an angle sufficiently different from 0° or 180°, and 2) displacement of the boundaries between domains with antiparallel or almost antiparallel spins.

Examining the reversible change in the magnetization dI_r in the general case we will assume that it can take place through both types of boundary displacement, i. e. that $dI_r = dI_{\perp r} + dI_{\parallel r}$, where $dI_{\perp r}$ is the reversible increment in the magnetization resulting from displacement of the boundaries between domains, the spins of which are not antiparallel, and $dI_{\parallel r}$ is the reversible increment in the magnetization caused by displacement of the boundaries between domains with antiparallel (or almost antiparallel) spins.

In iron crystals there are 6 and in nickel crystals 8 directions of easy magnetization. In weak fields the spins of domains are parallel to these directions. Therefore, in weak fields there are in iron crystals 6 and in nickel crystals 8 different magnetic phases. The volume concentrations of these phases will be denoted by $n_1, \ldots, n_i, \ldots, \bar{n}_1, \ldots, \bar{n}_i, \ldots$, where $\bar{n}_i$ denotes the concentration of the phase whose spins are antiparallel to those of the phase with concentration n_i. The general condition of equilibrium of the boundaries between two phases in a field H may be written in the form:

$$I_s H (h'_i - h'_k) = C_{ik}, \qquad (1)$$

where h'_i and h'_k are the cosines of the angles between the directions of the spins of phases i and k and the direction of the field H, I_s is the saturation magnetization. The magnitude C_{ik} is clearly independent of H and of the cosines h'. The left-hand side of (1) multiplied by dV represents the reduction of the magnetic energy; $C_{ik}dV$ represents the increment of the internal free energy when the volume of phase i is increased and that of phase k diminished by dV. If the change in the internal energy on displacement of the boundaries consists only in a change in the elastic energy owing to magnetostriction, for $i \neq k$, we obtain: $C_{ik} = -\lambda\sigma_s$, where σ_s is the mean value of the boundary tensile stresses in the direction of the spins of phase i, λ is the local magnetostriction when the direction of the spins is changed in one of the phases. When the direction of the field coincides with the direction of the spins of one of the phases, (1) takes the form of $HI_s = -\lambda\sigma_s$, *i.* e. is changed to the condition for equilibrium of the boundaries between domains, deduced by Bekker [5].

If one of the neighbouring phases is antiparallel, $h'_i = -h'_k$, and condition (1) assumes the following form[7]:

$$2HI_s h'_i = C_{ii}, \qquad (2)$$

For instance, the condition of boundary equilibrium between domains with antiparallel spins examined by Bloch [6] may be expressed in the form (2). In this case:

$$h'_i = 1; \; C_{ii} = \frac{1}{2a}\sqrt{\frac{K'a}{J}}\,\frac{\partial J}{\partial n} = \frac{1}{2a\delta}\,\frac{\partial J}{\partial n},$$

where a is the lattice parameter; K' is proportional to the constant of magnetic anisotropy and is of the same order of magnitude; J is the exchange integral; n, the normal to the surface of the boundary, and $\delta = \sqrt{\frac{J}{K'a}}$, the average width of the boundary layer between neighbouring phases according to Bloch. If it is assumed that $C_{ii}dV$ mainly represents the change in the elastic energy of the boundary zone when displaced condition (2) takes the following form [7]:

$$2HI_s h'_i = \frac{\lambda'}{2}\sqrt{\frac{J/a}{(K' + \lambda'\sigma_s)}}\,\frac{\partial\sigma_s}{\partial n} = \frac{\lambda'\delta'}{2}\,g, \qquad (2')$$

where λ' is a quantity of the same order as magnetostriction on saturation; $g = \frac{\partial\sigma_s}{\partial n}$; σ_s is the magnitude of the tensile stress along a line parallel to the direction of the spins of domains; a and K' are the same as before;

$$\delta' = \sqrt{\frac{J}{K' + \lambda'\sigma_s}}.$$

In calculating the reversible susceptibility we will commence with (1), the most general condition. Differentiating (1) by H, we obtain

$$I_s (h_i - h_k) = \frac{\partial C_{ik}}{\partial n} \cdot \frac{dn}{dH}; \qquad (3)$$

here h_i denotes the cosines of spins with the direction dH, dn is the

boundary displacement in a direction normal to the surface of phase i. If $\frac{\partial C_{ik}}{\partial n}$ and $\frac{dn}{dH}$ are of the same sign, displacement of any portion of the boundary when the field is increased by dH* will be reversible; if they are of opposite sign, an irreversible displacement of a portion of the boundary takes place. The average increment in the magnetization $(dI_{ik})_r$ in the direction dH during reversible displacement of the boundaries between phases i and k is equal to

$$(dI_{ik})_r = S_{ik} I_s (h_i - h_k) \overline{dn}, \tag{4}$$

where S_{ik} is the area of the surface of separation between phases i and k, $\overline{dn}$ is the mean reversible displacement of the boundaries in the direction of the normal to the surface of phase i. Averaging (3) and taking account of (4), we obtain:

$$\left(\frac{dI_{ik}}{dH}\right)_r = S_{ik} I_s^2 (h_i - h_k)^2 \overline{\left(\frac{1}{\frac{\partial C_{ik}}{\partial n}}\right)}. \tag{5}$$

If there is no grouping between the domains of definite phases i and k, it may be taken that the area of the surface of separation between phases S_{ik} is proportional to the product of their concentrations. We will represent S_{ik} in the following form:

$$S_{ik} = \gamma_{ik} n_i n_k, \tag{6}$$

where γ_{ik} may depend on both the structure of the material and the manner of obtaining the initial magnetic state (e. g. the state $I=0$, $H=0$). Equation (6) may not be considered valid for the phases with antiparallel spins. Theoretical calculation ([8]) shows that domains with antiparallel spins should become grouped in blocks. Therefore, the area of the surface of separation of phases with antiparallel spins may be correctly considered as proportional not to the product of the concentrations of these phases relative to the entire crystal, but to the product of their concentrations relative to the volume occupied by the phases in question multiplied by the sum of their concentrations relative to the entire volume of the crystal. We will accordingly represent S_{ii} in the following form:

$$S_{ii} = \gamma_{ii} \frac{n_i \bar{n}_i}{n_i + \bar{n}_i}. \tag{7}$$

We shall examine below the case of physical equality (equivalence) of all the directions of easy magnetization between themselves; in this case we have:

$$\overline{\left(\frac{1}{\frac{\partial C_{ik}}{\partial n}}\right)} = \overline{\left(\frac{1}{\frac{\partial C_{jl}}{\partial n}}\right)} = \begin{cases} C_1' \text{ for } i=k;\ j=l \\ C_2' \text{ for } i \neq k;\ j \neq l \end{cases} \tag{8a}$$

$$\gamma_{ik} = \gamma_{jl} = \begin{cases} \gamma_1 \text{ for } i=k;\ j=l \\ \gamma_2 \text{ for } i \neq k;\ j \neq l \end{cases} \tag{8b}$$

We will assume that the values of $C_1'\gamma_1$ and $C_2'\gamma_2$ do not change as the magnetization is changed**, i. e. as the concentrations of some phases are increased and that of others decreased.

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* If the directions of dH and H are different, h evidently does not coincide with h'.

** The assumption that C' is independent of I is equivalent to assuming the absence of any definite connection between the absolute magnitudes of C_{ik} and $\frac{\partial C_{ik}}{\partial n}$

Taking into consideration (6), (7) and (8) and summing (5) by i and k we obtain:

$$\sum_{i,k}\left(\frac{dI_{ik}}{dH}\right)_r=\chi_{2r}\sum_{i\neq k}\left\{(n_in_k+\bar{n}_i\bar{n}_k)(h_i-h_k)^2+(n_i\bar{n}_k+\bar{n}_in_k)(h_i+h_k)^2\right\}+$$

$$+\chi_{1r}\sum_i\frac{n_i\bar{n}_i}{n_i+\bar{n}_i}4h_i^2, \qquad (9)$$

where $i,k=1,2,3$ for iron, $i,k=1,2,3,4$ for nickel and

$$\chi_{2r}=I_s^2\gamma_2C_2';\ \chi_{1r}=I_s^2\gamma_1C_1'. \qquad (10)$$

After simple transformation the following formula for the reversible susceptibility is easily obtained from (9):

$$\chi_r=\chi_{\perp r}+\chi_{\parallel r} \qquad (11a)$$

$$\chi_{\perp r}=\chi_{2r}\sum_{i\neq k}\left\{(n_i+\bar{n}_i)(n_k+\bar{n}_k)(h_i^2+h_k^2)-2(n_i-\bar{n}_i)(n_k-\bar{n}_k)h_ih_k\right\} \qquad (11b)$$

$$\chi_{\parallel r}=\chi_{1r}\sum_i\left\{1-\left(\frac{n_i-\bar{n}_i}{n_i+\bar{n}_i}\right)^2\right\}(n_i+\bar{n}_i)h_i^2\text{*}. \qquad (11c)$$

When $I=0$, i. e. when $n_i=n_i$ formula (11b) turns into the formula for $\chi_{\perp 0}$ of the previous article [1]. Here, however, it should be noticed that the expression for χ_0 given in the previous article represents the initial susceptibility, whilst (11) for $n_i=\bar{n}_i$ represents only the initial reversible susceptibility χ_{0r} which is defined as the limiting value of the reversible susceptibility at $I=0$ and $H=0$. Even though for $n_i=\bar{n}_i$ (11) coincides with the expression obtained previously for χ_0 except that the multiplying factor is different, in some cases experiment shows that takes place $\chi_0>\chi_{0r}$ **.

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for the crystal as a whole. The assumption that γ is independent of I is equivalent to one or other of two mutually independent assumptions: either 1) the assumption of a certain uniformity of magnetization of the crystal, uniformity in the sense that the average magnetization of portions of the crystal in which the magnetic phases are more dispersed (i. e. in which the magnetic structure is finer) is approximately equal to the average magnetization of those portions in which they are less dispersed (in which the domains are «coarser»); or 2) the assumption of the absence of a definite connection between the dispersivity of the magnetic phases in different portions of the given crystal and the maximum values of the absolute magnitudes C_{ik} and C_{ii} in these portions. It should be remarked that the assumption made above that $\gamma C'$ is independent of I is obviously less exacting than the assumption that γ and C' are separately independent of I.

* It should be noticed that if in deducing (11) formula (6) were taken for S_{ii} instead of (7), i. e. if it were considered that $S_{ii}=\gamma_{ii}n_i\bar{n}_i$, where γ_{ii} is independent of $n_i\bar{n}_i$ (or independent of I, which is the same) we obtain the following expression for $\chi_{\parallel r}$ instead of (11c):

$$\chi_{\parallel r}=\chi_{1r}\sum_i\left\{1-\left(\frac{n_i-\bar{n}_i}{n_i+\bar{n}_i}\right)^2\right\}(n_i+\bar{n}_i)^2h_i^2. \qquad (11d)$$

** In this connection the author has reason to suppose that in pure crystals the case of $(\chi_{\parallel})_0>(\chi_{\perp})_0$, whilst $(\chi_{\parallel})_{0r}<(\chi_{\perp})_{0r}$, is possible.

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PHYSICS

THE MAGNETIC ANISOTROPY OF FERROMAGNETIC CRYSTALS IN WEAK FIELDS

By E. KONDORSKY

III. THE REVERSIBLE SUSCEPTIBILITY OF IRON CRYSTALS IN VARIOUS CRYSTALLOGRAPHIC DIRECTIONS

(Communicated by S. I. Wawilow, Member of the Academy, 17. III. 1938)

Formula (11) of the foregoing part may be used to calculate the reversible susceptibility of iron and nickel crystals in different directions. Below we shall give expressions for χ_r for crystals of iron as obtained from (11) for two extreme cases:

1) When phases whose spins form smaller angles with a line parallel to the direction of the field for all values of I predominate considerably over the other phases *.

2) For the case when the phase concentrations are equal in the initial state: $I=0$, $H=0$.

Kaya and Takaki's (1) and also apparently Williams' (2) crystals are examples of the first case. Crystals in which during treatment no prevailing tensil stress in one definite direction occurred (e. g. in the direction possessing the least demagnetizing factor **) may exemplify the second case.

1. Magnetic Field Is Parallel to the Direction [100]

In this case $h_1=1$, $h_2=h_3=0$, the magnetization $I=(n_1-\bar{n}_1)I_s$; whence

$$\left.\begin{aligned} &n_1-\bar{n}_1=j, \text{ where } j=\frac{I}{I_s} \\ &n_2-\bar{n}_2=n_3-\bar{n}_3=0 \end{aligned}\right\} \qquad (12)$$

* It should be noticed that if the concentrations of the magnetic phases are unequal in the initial state, the directions of easy magnetization can no longer be considered as physically equal, and equation (8) of the first part becomes inapplicable. However, in the limiting cases of unequal concentrations considered below, the concentrations of some of the phases are zero and those of the others equal themselves in the initial state. Equation (8) hold for the phases which remain, and formulae (9) and (11) of the first part are, therefore, applicable to the limiting cases in question.

** R. Janus and J. Shur (Ural Physico-Technical Institute) were right in pointing out to me that the formation of tensil stresses along the direction with the least demagnetizing factor in soft crystals may be the effect of the influence of the earth magnetic field produced during the thermic treatment.

a) The phases of spins which form smaller angles with a line parallel to the direction of the field predominate over the others.

Let us examine the limiting case of optimal orientation when all the spins are either parallel or antiparallel to the direction [100]. In this case:

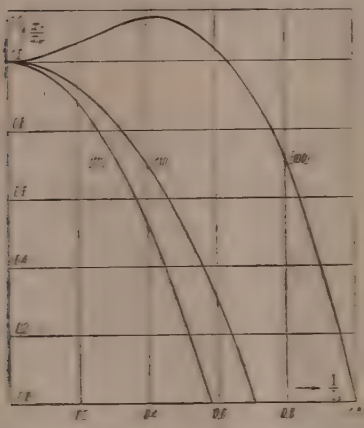
$$\left.\begin{aligned} n_1+\bar{n}_1&=1\\ n_i+\bar{n}_2&=n_3+\bar{n}_3=0 \end{aligned}\right\} \tag{13}$$

Substituting (12) and (13) in (11) we obtain

$$\left.\begin{aligned} \chi_{\perp r}&=0\\ \chi_{\parallel r}&=\chi_{1r}(1-j^2) \end{aligned}\right\} \tag{14}$$

Hence, if $\chi_{1r}>0$, it follows that:

$$\frac{\chi_r}{\chi_{0r}}=1-j^2. \tag{15}$$



Curves of the reversible susceptibility of a crystal of iron calculated for the case of equal concentrations of the magnetic phases in the initial state and for $\chi_{1r}=0$.

b) The concentrations of the magnetic phases are equal in the initial state, when $I=0$.

In this case a strict calculation of n_i is very difficult except the particular case when $I \parallel [111]$. In the general case n_i may be determined as a function of j from measurements of the magnetostriction only(1). An attempt may be made to calculate χ_r for the case of equal concentrations in the initial state by using Heisenberg's (3) formula for n_i obtained by statistical methods. The applicability of statistical methods to the calculation of n_i has as yet no theoretical basis, nevertheless experiment shows that values of n_i calculated by these methods are in satisfactory agreement with those determined from magnetostriction measurements in the case of not very soft crystals and polycrystals. [This does not, of course, apply to the case of non-uniform initial concentrations.]

Statistical calculation gives the following values of $n_i+\bar{n}_i$ for the case of magnetization parallel to the axis [100]:

$$\left.\begin{aligned} n_1+\bar{n}_1&=-\frac{1}{3}+\frac{j^2}{3}\sqrt{1+3j^2}\\ n_2+\bar{n}_2&=n_3+\bar{n}_3=\frac{2}{3}-\frac{1}{3}\sqrt{1+3j^2}. \end{aligned}\right\} \tag{16}$$

Substituting (12) and (16) into formula (11) of the first part we obtain:

$$\chi_{\perp r}=\frac{2}{9}\chi_{2r}\left(-4-6j^2+5\sqrt{1+3j^2}\right); \tag{17a}$$

$$\chi_{\parallel r}=\frac{1}{3}\chi_{1r}\frac{5+3j^2-4\sqrt{1+3j^2}}{-1+2\sqrt{1+3j^2}}. \tag{17b}$$

As in the previous case, the ratio $\frac{\chi_r}{\chi_{0r}}$ depends on the relationship between χ_{2r} and χ_{1r}. If $\chi_{1r} \ll \chi_{2r}$ it follows from (17) that if the magneti-

zation is increased within the limits of $0 < j < \frac{\sqrt{3}}{4}$ an increase takes place in χ_r (Fig. 1). If the function $\frac{\chi_r}{\chi_0}(j)$ is measured for iron in the direction [100], either (17) or (14) may be used to find the relationship between χ_{1r} and χ_{2r} and the question may be settled as to whether displacement of the boundaries between phases with antiparallel spins can in general cause reversible changes in the magnetization. (Measurements of the reversible susceptibility of ferro-nickel alloys under strong tension give reason to suppose that $\chi_{1r} \ll \chi_{2r}$.)

2. Magnetic Field Is Parallel to the Axis [110]

In this case $h_1 = h_2 = \frac{1}{\sqrt{2}}$; $h_3 = 0$. If the directions [100] and [010] are physically equal, we obtain from the condition: $I = \frac{1}{\sqrt{2}}(n_1 - \bar{n}_1 + n_2 - \bar{n}_2) I_s$ that:

$$\left.\begin{aligned} n_1 - \bar{n}_1 = n_2 - \bar{n}_2 = \frac{1}{\sqrt{2}} j \\ n_3 - \bar{n}_3 = 0 \end{aligned}\right\} \qquad (18)$$

a) Phases with spins forming smaller angles with a line parallel to the direction of the field predominate over the others.

Let us again examine the limiting case of

$$\left.\begin{aligned} n_1 + \bar{n}_1 = n_2 + \bar{n}_2 = \frac{1}{2} \\ n_3 + \bar{n}_3 = 0 \end{aligned}\right\} \qquad (19)$$

Substituting (18) and (19) in formula (11) of the first part we obtain:

$$\chi_{\perp r} = \frac{1}{4}\chi_{2r}(1 - 2j^2); \qquad (20a)$$

$$\chi_{\parallel r} = \frac{1}{2}\chi_{1r}(1 - 2j^2). \qquad (20b)$$

Hence, independently of the ratio between χ_{1r} and χ_{2r}

$$\frac{\chi_r}{\chi_{0r}} = 1 - 2j^2. \qquad (21)$$

b) The phase concentrations are equal in the initial state. When $H \parallel [110]$ statistical calculation gives:

$$\left.\begin{aligned} n_1 + \bar{n}_1 = n_2 + \bar{n}_2 = \frac{2}{3} - \frac{1}{3}\sqrt{1 - \frac{3}{2}j^2} \\ n_3 + \bar{n}_3 = -\frac{1}{3} + \frac{2}{3}\sqrt{1 - \frac{3}{2}j^2} \end{aligned}\right\} \qquad (22)$$

Substituting (18) and (22) into formula (11) of the first part we obtain:

$$\chi_{\perp r} = \frac{2}{9}\chi_{2r}\left(\frac{1}{2} - \frac{3}{2}j^2 + \frac{1}{2}\sqrt{1 - \frac{3}{2}j^2}\right); \qquad (23a)$$

$$\chi_{\parallel r} = \frac{1}{3}\chi_{1r}\frac{5 - 6j^2 - 4\sqrt{1 - \frac{3}{2}j^2}}{2 - \sqrt{1 - \frac{3}{2}j^2}}. \qquad (23b)$$

In this case, just as in the analogous case for $H \parallel [100]$, the ratio $\frac{\chi_r}{\chi_{or}}$ depends on the relationship between χ_{1r} and χ_{2r}. It should, however, be noticed that when $\chi_{1r} \ll \chi_{2r}$ the expression for $\frac{\chi_r}{\chi_{or}}$ obtained from formula (23) is practically very little different from expression (21).

3. Magnetic Field Is Parallel to the Axis [111]

In this case $h_1 = h_2 = h_3 = \frac{1}{\sqrt{3}}$. If the directions of easy magnetization are physically equal, the phase concentrations should be equal in the initial condition when $I = 0$. We then have:

$$n_1 - \bar{n}_1 = n_2 - \bar{n}_2 = n_3 - \bar{n}_3 = \frac{1}{\sqrt{3}} j; \tag{24}$$

$$n_1 + \bar{n}_1 = n_2 + \bar{n}_2 = n_3 + \bar{n}_3 = \frac{1}{3}; \tag{25}$$

Substituting (24) and (25) into formula (11) of the first part we obtain:

$$\chi_{\perp r} = \frac{2}{9} \chi_{2r} (1 - 3j^2); \tag{26a}$$

$$\chi_{\parallel r} = \frac{1}{3} \chi_{1r} (1 - 3j^2). \tag{26b}$$

Hence, again independently of the relationship between χ_{1r} and χ_{2r} we obtain:

$$\frac{\chi_r}{\chi_{or}} = 1 - 3j^2. \tag{27}$$

There will be divergence from (27) if the section of the specimen perpendicular to the direction of the magnetic field deviates considerably from the circular form and if the demagnetizing factor becomes either less or greater along one of the three axes of easy magnetization than along the others. Formula (25) may obviously be incorrect in this case. The above, of course, refers to the case of $H \parallel [110]$ too.

It should be noticed that expressions (14), (15), (21) and (27) remain valid if (11d) is used for $\chi_{\parallel r}$ instead of (11c), i. e. if we take $S_{ii} = \gamma_{ii} n_i \bar{n}_i$ instead of (7). It is easily seen from (11c) and (11d) that in the latter case the right-hand sides of (20b) and (26b) are multiplied by $\frac{1}{2}$ and $\frac{1}{3}$ respectively and the denominator in equations (17b) and (23b) becomes unity.

The expressions for χ_r can be obtained in an analogous manner for crystals of nickel. The reversible susceptibility can be calculated as a function of j for polycrystals of iron and nickel by averaging the general formula for χ_r. The theoretical formula obtained in this manner agrees quite well with experiment. It should be noticed that the theoretical curves of $\frac{\chi_r}{\chi_{or}}$ for polycrystals of iron and nickel are very close to one another; yet they do not coincide.

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PHYSIQUE DU GLOBE

OBSERVATION DE L'INTENSITÉ VARIANTE DE LA RAIE VERTE DANS L'ÉMISSION LUMINEUSE DU CIEL NOCTURNE

Par N. WASSMUTH, V. WERZNER, S. TIBILOV et S. FREIVERT

(Présenté par S. I. Wawilow, de l'Académie, le 25. III. 1938)

En été 1934 deux groupes d'une expédition scientifique de l'Académie à Elbrouz ont observé, l'un indépendamment de l'autre, le changement de l'intensité lumineuse de la raie verte dans l'émission du ciel nocturne.

Pour isoler la raie verte ($\lambda = 5577.3$ Å) le groupe de l'Institut Optique se servait d'un monochromateur et comparait l'intensité de la raie étudiée avec celle du spectre lumineux continu du ciel nocturne dans la région $\lambda = 5595$ Å,

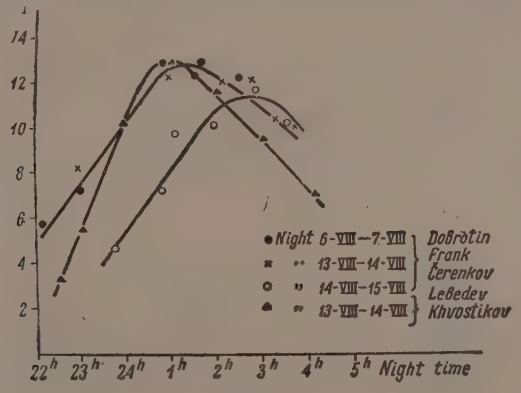

Fig. 1.

tandis que le groupe de l'Institut Physique faisait usage du filtre proposé par Rayleigh. Dans ce dernier cas l'intensité mesurée était comparée avec celle d'une source constante, notamment un cristal de sel uranique à émission spontanée.

On photométrait la raie verte d'après la méthode d'extinction.

Une croissance de l'intensité a été observée, qui atteint son maximum entre 1 h. et 2 h. 30 m. du matin. On le voit nettement sur les courbes en fig. 1 qui présentent les observations faites par les deux groupes. Le nombre d'observations étant restreint, il était alors impossible de rattacher ce maximum à un temps déterminé de la nuit, non plus affirmer qu'il change sa position de jour en jour.

Donc, le groupe optique, prenant part à une autre expédition de l'Académie à Elbrouz, en 1937, s'est proposé au cours de son travail principal de vérifier et déterminer plus exactement la position du maximum de

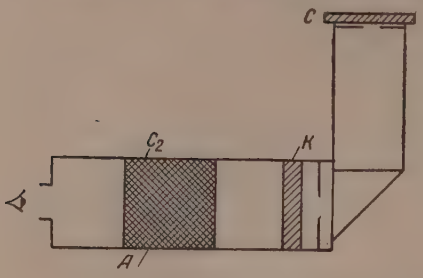

Fig. 2.

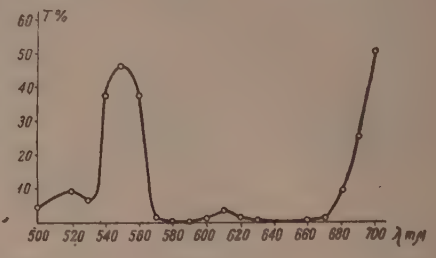

Fig. 3.

l'intensité de la raie verte ainsi que d'établir sa valeur relative. Le photomètre dont on a fait usage en mesurant l'intensité de la raie par la méthode d'extinction, est représenté en fig. 2 schématiquement. La

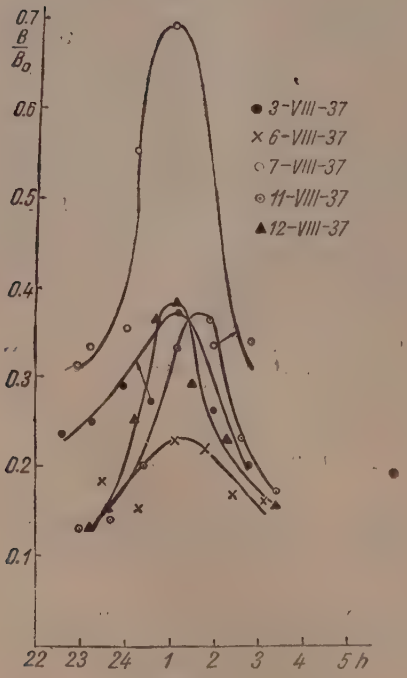

Fig. 4.

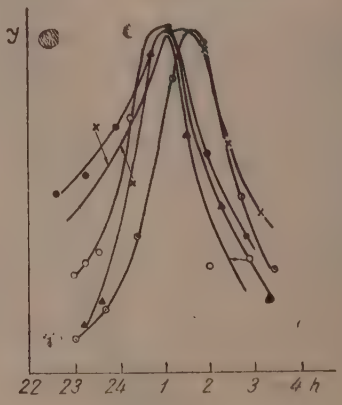

Fig. 5.

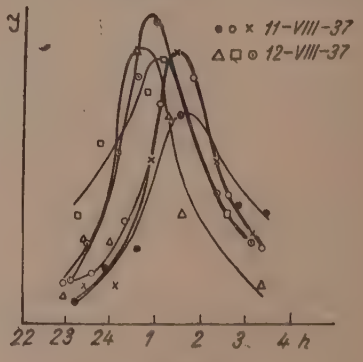

Fig. 6.

lumière de la région photométrée du ciel (environ l'Étoile polaire), en traversant les filtres C_1 et C_2 (dont la transmission spectrale est représen-

tée en fig. 3), était affaiblie par le coin neutre K jusqu'à une intensité correspondant au seuil visuel de l'observateur dont l'oeil avait été adapté à l'obscurité. Le diamètre angulaire du champ visuel égalait 5°.

L'intensité lumineuse de la région photométrée du ciel nocturne était comparée avec l'intensité constante de l'étalon, ce qui permettait d'éliminer le seuil visuel de l'observateur.

Comme dans les années précédentes, on avait choisi pour lieu d'observation le «Krugozor» de l'Elbrouz situé à une altitude de 3000 m (43°17′ lat. N.; 42°28′ long. E.).

Fig. 4 présente les courbes de variation de l'intensité lumineuse de la raie verte au cours de la nuit, observée le 3, 6, 7, 11, 12 Août, 1937. Les abscisses indiquent l'heure moyenne locale tandis que les ordonnées, l'intensité de la raie verte par rapport à celle de l'étalon.

Les courbes confirment d'une manière évidente l'existence d'un maximum. En outre on voit sa valeur changer de jour en jour.

Ci-dessous sont données des valeurs du maximum de l'intensité lumineuse de la raie verte par rapport à l'étalon dont l'intensité restait constante au cours de l'observation.

3.VIII.1937	0.37
6.VIII.1937	0.23
7.VIII.1937	0.69
11.VIII.1937	0.36
12.VIII.1937	0.40

Les courbes montrent de plus que la position du maximum n'est pas la même à de différents jours, ce qui est d'accord avec les observations de 1934 dont les données sont présentées en fig. 1.

Pour simplifier la comparaison, la valeur du maximum dans les courbes de la fig. 5 est prise égale à unité. Le maximum change de position entre 1 h. et 1 h. 45 m. du matin.

Pour être sûr que le déplacement du maximum n'est point à imputer à un mesurage incorrect, plusieurs observateurs pendant deux nuits ont mesuré l'intensité simultanément et indépendamment l'un de l'autre.

Les résultats de leurs observations sont présentés en fig. 6. On voit bien que, dans les limites de l'erreur d'observation, les positions du maximum sont identiques chez les trois observateurs.

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GENETICS

DYNAMICS OF THE GENIC COMPOSITION IN WILD *DROSOPHILA MELANOGASTER* POPULATION

By J. OLENOV and I. KHARMAC

(Communicated by N. I. Vavilov, Member of the Academy, 20. III. 1938)

The concentration of mutant genes in wild populations of *Drosophila melanogaster*, though occasionally very high, is subjected to considerable yearly variations [Dubinin and associates (1, 2, 3)]. Upon this fact a number of theoretical speculations have been based. However, to obtain evidence of the relative importance of each of the factors influencing the genic composition of the population we considered it necessary to examine the changes which in the course of one season, at the period when it is most numerous, occur in the population. The present paper deals with the data obtained on this score.

For our investigations we chose a population secured from a factory of fruit products in the town Uman (Ukraine, Kiev region). Within the period from end July 1937 to end September of the same year the population was very numerous, amounting to no less than 200—300 thousands specimens. The concentration of the mutant genes in chromosome II was examined after the $\frac{Cy\ Sp}{L^2}$ method. Flies caught on August 3—18 served as material for sample I. The subsequent sample (AI, i. e. autumnal I) was caught on September 11—13. On September 22 pupae were collected in the factory and the flies which hatched from them between the 23 and the 25 of September were likewise examined (sample K). The purpose was to bring into evidence the changes which occurred in one generation (the duration of a developmental cycle was from 11 to 12 days). At last on September 23—25 the imago were caught on the factory (sample AII).

No marked effect in the genic composition of the population could be produced neither by immigration nor emigration, the population being very numerous. It may be mentioned here that according to our data in places, lying at a distance of from 3 to 5 km from the habitat investigated, the same genes as in the factory population and in similar concentration were found. The genic composition of the population examined seems to have reflected the characteristic features of the populations of the entire district in the autumn of 1937.

Considered as a whole, the concentration of lethal and visible mutations did not lower throughout investigation. Inversely, it is registered a certain tendency to the increase (see the table). However, conclusions basing on this fact may only be made if the concentration of each separate mutation is known, since it is by the concentration of the mutation that it is possible to judge of the chances for this mutation to appear in a homozygous state and, consequently, to undergo a more intensive natural selection. An identification of the 23 lethals found in flies secured on the 3rd of August (sample I) has shown that in only one of the lethal genes the concentration is above

Sample	Number of chromosomes examined	Number of lethals	Lethals (percentage)	Number of visible mutations	Visible mutations (percentage)
I	237	46	19.4±2.6	14	5.9±1.7
AI	461	97	21.4±1.9	24	5.2±1.0
AII	526	120	22.8±1.8	27	5.1±1.0
K	389	94	24.2±2.2	27	7.0±1.3

Diff. I—AII by lethals . . . . . 3.4±3.2
Diff. AI—K by lethals . . . . . 2.8±2.9
Diff. I—K by lethals . . . . . 4.8±3.4

2%, the majority of genes being of much rarer occurrence. Neither had any of the visible mutations a rate of occurrence markedly higher than 1%. The low concentration of all mutant genes contained in the population excludes the possibility of an effective action of natural selection by homozygotes. However, two other factors capable of producing changes in the genic constitution of the population remain to be considered: the mutational process and selection by heterozygotes which depends upon the different viability or fertility of the heterozygotes as compared to the normal individuals.

The investigation of the frequency of mutations in chromosome II was carried out as follows. Several $\frac{Cy\ Sp}{II\ n}$ males (II *n*—the normal natural chromosome) were taken from 17 stocks of the sample AII—which accordingly to previous analysis were found to be free of mutational changes—and their offspring examined by the $\frac{Cy\ Sp}{L^2}$ method. On the whole 451 chromosomes—from 60 males—were examined. As from each male several chromosomes were studied, all the cases in which a mutation had appeared in one of the previous generations could easily be brought into evidence and omitted from final calculation. In this experiment 19 lethal mutations were registered; upon identification some of them proved to be arisen 14 times. The percentage of gametes which contained lethal mutation in chromosome II originated in one generation is 4.2±1.0. (The higher frequency of lethal mutations in chromosome II observed in our population was not due to any special breeding conditions, since in the flies investigated at the same time in our laboratory, which belonged to the laboratory stock *white*, 9 lethal mutations were established for 877 chromosomes, i. e. 1.0±0.3%.) Moreover, in this experiment several visible mutations had arisen.

Admitting that the observed frequency of mutations expresses approximately the frequency of mutations occurring in chromosome II in a population under natural conditions, the conclusion would be that within 45 days (the period of time over which our investigations extended) the mutational process should have considerably heightened the concentration of the mutant genes. Further, according to our data certain mutations in the population investigated are under certain conditions liable in their heterozygous state to positive natural selection. Thus, e. g. it was established by special experiments that under conditions of overpopulation the cross-

ing $\frac{II\ n}{II\ n} \times \frac{Cy\ Sp}{II\ n}$ produces in F_1 51.9±0.8% *Cy* flies, whereas under the same conditions the crossing $\frac{II\ n}{II\ n} \times \frac{Cy\ Sp}{MS}$ (*MS* is a visible mutation involving eyes and wings) produces 32.5±1.1% *Cy* only (*Diff*=19.4±1.3). Consequently, the tendency to increase the concentration of mutant genes observed in the population is easily explained by the mutational process and perhaps by the positive selection of certain genes in heterozygous state.

The importance of the mutational process as a factor determining the genic composition of the population may be shown in the following way too. All the visible mutations which have arisen in the experiment described above, the aim of which was to establish the frequency of mutations in chromosome II, were found to be identical to those observed under natural conditions. The 14 lethals arisen in the same experiment were identified with the 23 lethals found in flies secured on the 3rd of August (sample I) and two, that is about 15%, proved to be identical. As the population contained numerous lethals, each of them being present at a very low concentration, this identity was equally very characteristic. We have consequently all reason to believe that qualitatively the genic constitution of the population is determined by the mutational process; it is, however, necessary to take also into account the possible influence of selection by heterozygotes.

Of all visible changes observed in the population the most frequent was an altering of the normal venation in the wing, involving the crossvein posterior. In various lines this vein was either absent or did not reach the longitudinal veins or else it was duplicated, while in some cases it gave off a branch. Such incomplete change (or, more exactly, changes) was observed in 16 out of 1613 chromosomes, i. e. in 1% of the cases. While studying other visible mutation, a very curious fact came to our observation. It was found that in 19 out of 92 lines containing various visible mutations, that is in about 20% of the cases, the main manifestation of the different genes is accompanied by similar disturbances involving the crossvein posterior. Further observations on this point were made while mustering flies diheterozygous in respect of natural lethals, which had bred from crosses undertaken for the identification of lethals. Non-*Cy* flies were mustered in 81 crosses and in 17 instances the same alterations in the venation of the wing were found. This characteristic feature occurs also in monoheterozygotes in respect of lethals, but it is less frequent and is less strongly expressed. From the data above it may be concluded that the disturbances involving the crossvein posterior appear as a result of various alterations of the genotype of the flies in the population investigated. The main gene which produces this alteration in the venation must be widespread among the population. Natural selection while eliminating the harmful effect of this gene renders it incapable of independent manifestation of characters open to investigation, and we may observe the process of its inclusion into a normal genotype. Assuming this interpretation of the phenomena observed to be true, we may expect that in the 16 lines—in which the character discussed has been noted as the only change subject to registration and not as a peculiarity accompanying the main manifestation of their genes—this character may also be accounted for by differing genes. It is evident from the above that there must exist numerous modificators capable of activating the manifestation of the main gene. The identification of these 16 lines is rendered difficult because in some of them the character is not fully dominant and is not fully manifest. The degree of dominance is likewise various—this may be observed in crosses of the same line with stocks containing different normal

wild chromosomes II (II *n*1, II *n*2, etc.). Nevertheless the results of identification are unequivocal: most crosses gave phenotypically normal offspring.

The case discussed is, we should say, an instance of the effect of the action of natural selection which includes a mutant allelomorph in the normal genotype. This example makes it possible to inquire more deeply into the process which has been postulated by the genetic theories of natural selection.

Besides the study of chromosome II an analysis of 1712 *X*-chromosomes was carried out. Of the 856 females examined 8 proved to be heterozygous with respect to sex linked visible mutations. Two wing mutations and one mutation shortening the bristles were established. The remaining 5 females were heterozygous in the *y* gene. In the same experiment three cases of visible mutation were established. In two cases the mutations arose in the locus *y* and in one case the mutation occurred in the normal allelomorph of the gene shortening the bristles (it proved to be identical to the mutation found under natural conditions). With the view of comparing the frequency of mutations in chromosome II of the flies in our population with the frequency of mutation in the *X*-chromosome 837 *X*-chromosomes were examined after the *ClB* method. In this experiment 10 lethal mutations arose which makes up $1.2 \pm 0.3\%$. (In the *X*-chromosomes of the laboratory lines examined at the same time at our laboratory—376 *X*-chromosomes of the *Florida* and 645 *X*-chromosomes of the *y* stock—1 lethal mutation was discovered which makes up $0.1 \pm 0.1\%$.) Thus, the high frequency is characteristic not only of chromosome II of the population investigated, but also of the *X*-chromosome. It should be noted too that in this experiment likewise mutations in the locus *y* have been several times registered.

Among the genes of chromosome II, the gene of divergent wings showed a high mutability; this mutation has repeatedly arisen in such cultures where the gene was sure to be represented by its normal allelomorph. Other genes of chromosome II of the investigated population mutate also rather frequently. Even leaving aside the changes involving the crossvein posterior, we still find that in the experiment just described in 451 chromosome II, 5 visible mutations are fully manifesting themselves (twice we found mutation of normal allelomorph of the divergent wings gene).

As had previously been stated, the genic constitution of the population depends qualitatively on the process of mutation. The automatic-genetic processes when producing quantitative changes in the genic composition of the population by reason of the nature of their action must in most cases heighten the concentratoin precisely of those mutant genes which, owing to the process of mutation and selection by heterozygotes, have already reached considerably high concentration. Also it is the genes which have reached a high concentration that of all the mutant genes are the most likely—all other conditions being equal—to be included through natural selection into the normal genotype.

Therefore we consider the main problem of further investigation to consist alongside of the investigation of various forms of natural selection and its intensity in an analysis of the causes accounting for the quantitative and qualitative peculiarities in the mutability proper to some wild populations as compared with others.

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AGROPHYSIOLOGY

BEHEADING OF SUNFLOWER AS A MEANS OF INCREASING THE YIELD OF ITS VEGETATIVE MASS

By G. ZABLUDA

(Communicated by A. A. Richter, Member of the Academy, 28. III. 1938)

In the northern localities of the Soviet Union the extension of sunflower crop for silage meets with the absence of local seed grains valuable for the purpose, because the high yield silage varieties of sunflower do not get mature in the North. To overcome the difficulty one should look for various ways of increasing the yield of vegetative mass in some edible and oil yielding varieties which ripen under conditions prevailing there. This attained one has solved the problem of green crop for silage and of local sowing material.

In approaching the problem we must above all take into consideration that the flowering stage is a critical point in the development of annual plants after which the ageing and dying away of separate vegetative organs sets in([1]). This has been confirmed for sunflower by the observations made at the Urupsk test-station([2]), where it has been established that with the outset of flowering stage the sunflower is far from increasing the bulk of green crop and even lowers it sometimes owing to ageing and dying of lower leaves.

Aside from direct loss due to the fall of lower leaves the ageing of plants is attended by a gradual, both relative and absolute decrease of valuable nutritive components of the green mass and a respective increase of hardly digestible substances([3]). All the aforesaid is the reason why it is recommended to harvest sunflower for silage when it is passing or, even better, beginning ist flowering stage([4]). The latter thus is in the case considered a natural barrier for further storing of valuable green mass which is the object of our endeavour when cultivating sunflower for silage.

On the other hand, it is known that a removal of flower buds in annual plant protracts the period of vegetative growth ([5],[6]). An artificial interruption of flowering results in a greater yield of green mass.

The above facts suggested the idea of an artificial interruption of growth and development of inflorescences as a means of increasing the bulk of green crop both qualitatively and quantitatively. In field conditions and with respect to sunflower this may be achieved by removal of heads when they have begun forming. Such an experiment was carried out in 1937 on a test-field of the Agricultural Institute of the Chouvash Autonomous SSR.

The sort Selenka was available for the test. Planting was done on May 25. The plants stood 30 per square meter. The control plot had its plants with the head until harvesting, while in plants on the test plots the heads were nipped off as soon as they had begun forming. Harvesting was done on August 20, the control plants being in full bloom at that time.

Table 1

Height and Colour of Separate Organs of Plants at Harvesting

Set of plants \ Plant organs	Stems			Leaves		Heads		
	Average height in cm	Average diameter in cm	Colour	Average area in cm^2	Colour	Number per plant	Diameter in cm	Colour
Controls	140	0.8	Yellow-green	1080	Light green	1	8.5	Light green
Experiments	121	1.0	Intense green	1460	Dark green	3.5	2.0	Dark blue-green

Table 1 shows the removal of heads to result in a more intense green colouring of plants which is an evidence for the change in the dynamic of physiological processes. The stems grew lower in experiments than in controls but they were thicker and gave lateral shoots on which heads formed attaining 1—3 cm in diameter at the time of harvesting. Special attention should be payed to the increase of leaf area which undoubtedly enhanced the daily yield of the photosynthesis and told upon the bulk of the crop.

Table 2

Yield (in g) Gathered from 1 m^2

Plant organs \ Set of plants	Controls		Experiments			
			Raw bulk		Air-dry bulk	
	Raw bulk in g	Air-dry bulk in g	in g	% to control	in g	% to control
Heads	1235	208	533	43.2	96	46.2
Leaves	810	206	1210	149.4	303	147.1
Stems	1955	558	3045	155.8	790	141.6
Total bulk	4000	972	4788	119.7	1189	122.3

From Table 2 it may be seen that the raw (or air-dry) bulk yielded by test-plants is 19.7% (resp. 22.3%) greater compared to controls. The quantitative increase of the yield is attended by its qualitative modification.

Table 3

Chemical Composition of Various Organs of Plant

Set of plants	Plant organs	In % to air-dry matter						
		Water	Ash	Cellulose	Raw fat	Raw protein	Nitrogenless extract. subst.	Total
Controls	Heads	7.18	8.81	20.70	9.90	11.44	41.97	100
	Leaves	9.62	4.36	13.72	1.91	11.10	59.29	100
	Stems	6.66	12.12	18.61	2.21	10.25	50.15	100
Experiments	Heads	6.77	5.76	15.78	14.90	11.44	45.55	100
	Leaves	8.28	4.79	10.55	1.91	12.44	62.03	100
	Stems	7.40	10.37	13.36	2.13	10.56	56.18	100

In Table 3 the data of chemical analyses are given. They show how great is the influence of beheading upon the chemical composition of separate organs. From the high content of nitrogenless extractive substances and raw protein and from the low content of cellulose we may conclude that an artificial interruption of flowering not only protracts the period of vegetative growth but also removes the causes of plant ageing, no matter if the latter effect is a temporary one.

Table 4

Separate Components of Yield Gathered from 1 m²

Set of plants	Absolutely dry matter in grams					
	Ash	Cellulose	Raw fat	Raw protein	Nitrogenless extract. subst.	Total
Controls	95	175	36	104	490	900
Experiments	101	152	36	135	675	1099
Ratio of experiments to controls	106.3	86.8	100	129.8	137.7	122.1

To make this more obvious the quantitative data on the yield gathered from one square meter are given in Table 4 with reference to separate components. The table shows the quantity of raw fat to be the same in experiments and controls; the quantity of cellulose in experiments is 13.2% less while the quantity of nitrogenless extractive substances is 37.7% more and that of raw protein 29.8% more than in controls.

Following conclusions may be drawn from the above:

1. An artificially interrupted flowering of sunflower, by beheading the latter when the head starts forming, results in appearance of lateral shoots, increase of leaf surface, protracted period of vegetative growth, and a temporary elimination of causes producing the ageing of plants.

2. As a result of morphological and physiological changes the bulk of green crop is increased and the latter shows a lower content of cellulose and a higher content of nitrogenless extractive substances and raw protein.

3. The removal of heads by means of nipping or cutting is easy to be done on a large scale. This procedure, alongside of the other measures, may be turned to account in practice for the sake of increasing the silage crop of sunflower.

4. Aside from its practical value, the question discussed is of a great theoretical interest, as being concerned with the study of the dynamics of biochemical processes which take place in a plant passing from vegetative growth to reproductive development.

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PLANT PHYSIOLOGY

THE EFFECT OF PRELIMINARY TREATMENT WITH COLOURED LIGHT ON THE DEVELOPMENT OF PEANUT (*ARACHIS HYPOGAEA* L.)

By N. GORTIKOVA

(Communicated by A. A. Richter, Member of the Academy, 23. III. 1938)

The question as to the part played by spectral composition of light in the photoperiodism is since recently more and more attracting the interest of the experimenters. From the standpoint of method the works concerned with the question may be divided into two groups. To the first group belong those in which the authors are endeavouring to grow the plants under additional illumination with coloured light throughout their development. Monochromatic light of high intensity, as necessary for photosynthesis, being difficult to obtain, the authors added coloured light to white light either simultaneously, by intensifying the rays of the respective spectral region, or in succession, i. e. after the plants were given their daily rate of white light for several hours they received coloured light during several hours too. An example of such a method is afforded by the work of Katunsky (1937) who came to an interesting conclusion, viz. that with respect to photoperiodism the rays of different wave length are endowed with the same efficiency as is displayed by them in the photosynthesis, the green rays being the least efficient.

In the works of the second group the method was essentially different. The plants were subjected to preliminary treatment, their germinating seeds being given the coloured light for several days. Plants thus induced with the effect of coloured light were then grown under natural daylight.

This method was successfully applied by V. P. Malčevsky (1936) and A. A. Kuzmenko (1936). The first of them gave coloured light to the seeds which were normally germinating, the second illuminated the seeds whose germination was inhibited by a restricted water supply. Both authors are unanimous in that a red light induction accelerates the development.

These works, however, are lacking well defined data as to the effect of shorter wave rays of the spectrum. To supply this want we undertook our experiments with peanut whose seeds are distinguished by an abundant store of nutritive substances.

The germinating seeds put into conditions of sufficient water supply were given coloured light of different wave length during 10 days at a temperature varying within 25—30° C. As a source of light we used daylight which was passed through coloured glasses tested beforehand spectroscopically. To isolate the separate group of rays with sufficient accuracy we were compelled to take comparatively dark glasses which weakened the intensity of the colou-

red light; the seedlings had therefore chiefly, if not exclusively, to gain their nutriment from the seed. In 10 days they attained a height of 10—15 cm and were then transplanted into pots where they grew under daylight from June 1 to October 10, i. e. until harvesting. Previous to transplanting a portion of control leaves were tested for the content of chlorophyllines and their attendant carotinoids. The data of analysis are tabulated below. The figures presented in the table have been averaged from 8 determinations for each variant.

	1	2	3	4	5	6	7
	Control daylight	Red glass-light 700—610 mμ	Orange glass-light 620—500 mμ	Green glass-light 550—490 mμ	Blue glass-light 500—440 mμ	Dark-blue glass-light 480—430 mμ	Violet glass*-light 460—410 mμ
Grams of pigment per 100 g fresh weight of seedling leaves							
Chlorophyll	0.1200	0.0510	0.0031	0.0014	0.00058	0.0210	0.0021
Xanthophyll	0.0880	0.0710	0.0850	0.0230	0.00900	0.0540	0.0100
Carotin	0.0520	0.0220	0.0120	0.0099	0.00700	0.0210	0.0087
Total of pigments	0.2600	0.1440	0.1001	0.0343	0.01658	0.0960	0.0208
Number of days from planting to the outset of flowering							
	40	37	37	39	40	38	39
Number of leaves, flowers and fruits per plant at the time of harvesting							
Leaves	22	31	28	16	14	20	10
Flowers	13	34	21	9	3	11	8
Fruits	7	17	9	7	0	5	3
Dry weight of separate organs per plant at the time of harvesting							
Roots	1.00	1.20	1.00	0.60	0.30	0.66	0.28
Stems	1.60	2.10	1.80	1.00	0.80	1.80	0.80
Leaves	5.00	6.20	5.00	3.20	2.20	3.80	1.80
Flowers	0.04	0.35	0.05	0.01	0.01	0.06	0.02
Fruits	0.52	1.21	1.00	0.20	0.00	0.41	0.15
Gynophores	0.41	1.00	0.83	0.07	0.00	0.36	0.05
Whole plant	8.57	12.06	9.68	5.08	3.31	7.09	3.10

Comparing the tabulated figures it may essily be seen that a 10-day illumination of germinating seeds with coloured light exerts strong influence upon the subsequent development of plants. On the one hand, it has been observed an acceleration of flowering and a strong stimulation of vegetative and reproductive development after a treatment with red and orange-yellow light of a wave length ranging within 700—500 mμ. On the other hand, a shorter wave length has produced obvious depression of both reproductive and vegetative organs while a slight acceleration of flowering has been observed under the screens 4, 6 and 7. An intermediate position is occupied

* Dark-blue and violet glasses transmitted extreme red rays.

by the plants induced with white light. They lag with respect to flowering behind the plants induced with red and orange light and exceed the plants treated with green, blue, dark-blue and violet light in their vegetative and reproductive development. To explain this result we have to assume that the stimulating effect of red and orange rays, present in white light, was weakened by an opposite effect of rays with shorter wave length.

As seen from the table the plants induced with blue light developed most poorly (screen 5); they did not form fruits nor gynophores. The development of reproductive organs is obviously supressed by this group of rays.

The influence of dark-blue and violet rays is more favourable for the development of the plant, especially for its reproductive organs. It is a matter of regret that the glasses we used were not absolutely monochromatic; dark-blue and violet glasses transmitted extreme red rays. On this account the data of our experiment are not to be used for definite conclusion on the action of dark-blue and violet rays. The green rays (screen 4) are better than the blue ones as regards their influence upon the reproductive development.

Generally the data obtained bear out the statement advanced by Katunsky: the effect of monochromatic light upon the rate of development of the plant as a whole and of its reproductive organs is the same as in photosynthesis; in both cases the maximum falls within wave length of the less refrangible half of the spectrum. As to the effect of more refrangible rays the data of our experiment point to the presence of such a group of rays as causes marked depression of reproductive development. This group is not isolable with the screens used in experiment; it seems to contain part of green and blue rays (about 520—480 mμ).

One cannot help directing attention to the process of greening in coloured light. Here too, we see the same group to manifest its negative influence (screen 5); the ratio $\frac{\text{chlorophyllines}}{\text{carotinoids}}$ attains its maximum under these rays.

Thus, in studying the effect of the spectral composition of light on the photoperiodic response of the plants one should take into account that a group of rays is endowed with negative influence. A more precise determination of the wave length of these rays is the task of our further experiments.

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PALEONTOLOGY

NEW DATA ON *RHIPIDOCYSTIS* JKL (ORDER DIGITATA n. o., CLASS CARPOIDEA JKL) AND ON A NEW GENUS *BOCKIA* (SUBCLASS EOCRINOIDEA JKL, CLASS CRINOIDEA MILL.) FROM THE ORDOVICIAN OF LENINGRAD PROVINCE, USSR, AND ESTONIA

By R. HECKER

(Communicated by A. A. Borissiak, Member of the Academy, 22. III. 1938)

In 1900 Jaekel stated in the Ordovician of Leningrad province the presence of a new genus of Carpoidea—*Rhipidocystis* [Jaekel (2)]. The later works, namely, the monograph on Echinoderma by Jaekel(3) and Bather's work (1) have added nothing essential to the description of this form.

The studying of all the material available on *Rhipidocystis* Jaekel from Leningrad province and Estonia, which is now rather abundant, showed that under this name a whole number of forms is united, which as a matter of fact do not belong to one genus or even family, but to several classes and even to both sub-types (Pelmatozoa and Eleutherozoa) of Echinoderms. The genus *Rhipidocystis*, as conceived by Jaekel, comprises remains of four different genera, of which two belong to the class Carpoidea, one represents a form intermediate between the classes Cystoidea and Crinoidea, and the fourth one refers to the class of mobile Echinoderms—Ophiocistia. Thus, the «cup-plates» of this genus are parts of skeleton of a new Ophiocistia *Volchovia* and partially of a new Crinoid *Bockia*; the «stem-plates» belong to the Carpoid *Dendrocystites*; the flat «appendages of the stem» refer to the Carpoids, for which I retain the name *Rhipidocystis*; while the «sac-like roots» are cups of some other members of *Bockia*.

I. Diagnosis of the genus *Rhipidocystis* in its new interpretation: Cup flat, elongated-oval, with blunt anterior and tapering posterior ends. The marginal plates limited in number, have strongly thickened external margins, forming a massive peripheral circlet, usually very distinct from the median part of the cup; the median parts of the marginals, and the few centrals are thin. On the anterior end of the cup two rows of strongly developed long digits are observed, situated one above another; skeleton of the digits is composed by brachial and covering plates over the ambulacral furrow. On the anterior end of the cup there are observed one or two small apertures, besides the oral opening. The stem is rudimentary. Ordovician of Leningrad province and Estonia, from Glauconite (B_{II}) up to Jeve (D_1) horizons.

The genus *Rhipidocystis* Jkl includes 5 species, with following chief characters and stratigraphic position:

1. *Rhipidocystis baltica* Jkl. Cup up to 5.8 cm high; it has massive peripheral circlet, showing no swellings, up to 2.5 mm wide. From the lower surface of the anterior end of the cup derive 4—5 digits. The surface of the peripheral circlet is densely and fine granulated, the surface of the median

parts of marginals and of the centrals being smooth. Abundant in Glauconite limestone (B_{II}), more sparsely in Orthoceratites limestone (B_{III}) of Leningrad province.

2. *Rhipidocystis batheri* n. sp. A large form, with massive peripheral circlet up to 4 mm wide. The whole surface granulated; the granulation being finer than that of the peripheral circlet of *Rh. baltica*; on the peripheral circlet sparse coarser granules are scattered amid the smaller ones. Echinosphaerites limestone (C_1) of Leningrad province.

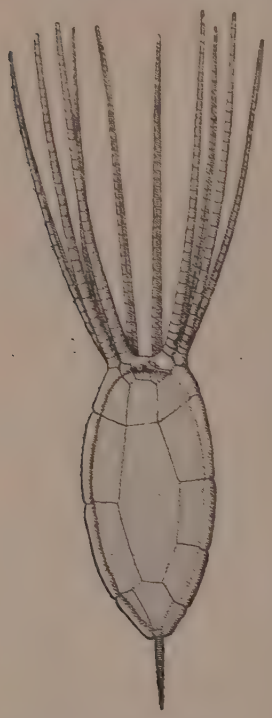

Fig. 1. Reconstruction of *Rhipidocystis* Jkl.

3. *Rhipidocystis esthona* n. sp. Plates of the peripheral circlet form a considerable thickening when meeting on the posterior end of the cup. The surface of the circlet is densely and fine granulated. Kuckers horizon (C_2) of Estonia.

4. *Rhipidocystis öpiki* n. sp. Height of the cup 5 cm. Differs from *Rh. baltica* by the presence of a greater number of digits (apparently 6) on the lower surface of the cup. Digits arranged in two rows, 3 of them in each, their length exceeding 3 cm. The surface of the brachial plates of the digits of the peripheral circlet and, apparently, of the median parts of marginals and of the centrals too is densely granulated, the granulation being finer than in *Rh. baltica*, coarser and denser than in *Rh. batheri*. Jeve horizon (D_1) of Estonia.

5. *Rhipidocystis robusta* n. sp. A massive, thick-walled, broad form. On the posterior end of the cup the peripheral circlet forms a strong thickening like that in *R. esthona*. The circlet passes into median parts of marginals gradually, without constriction. It is covered by fine granulation with scattered coarser granules which are larger than in *Rh. batheri*. Surface of the median parts of the plates smooth. Jeve horizon (D_1) of Leningrad province and Estonia.

The genus *Rhipidocystis* cannot without qualification be referred to the class Carpoidea. This genus displays the following essential characters of Carpoids: 1) the absence of radial symmetry; 2) an elongated flat form of the cup; 3) the presence of a massive peripheral circlet; 4) no pores of Cystoid type; 5) poorly developed apertures of the body; 6) a rudimentary stem. At the same time, by the development of strong articulated appendages of the ambulacral system, the form under consideration differs from all the known forms of Carpoids which, at the most, possessed only weakly developed appendages or a single «brachiole». The absence of appendages was considered up to now as one of the typical characters of Carpoids, and was included into the diagnosis of this class. Now this diagnosis must be correspondingly altered.

Furthermore, the genus *Rhipidocystis* Jkl—as defined above—as well as the family *Rhipidocystidae* Jkl, have to be excluded from the order Soluta Jkl, the diagnosis of the latter having been correspondingly altered. Only one family *Dendrocystitidae*, including the genera *Dendrocystites* Barr., *Dendrocystoides* Jkl and *Syringocrinus*, will remain in the order Soluta. As *Rhipidocystis* can be placed in none of the remaining three orders—Cincta Jkl, Mitrata Jkl and Cornuta Jkl—it appears expedient to erect for it a new order, to which I give the name Digitata.

Diagnosis of the order Digitata nov. ord.: Cup flat, with parallel upper and lower sides; its massive peripheral circlet is formed by distal borders of the marginals only, while its median part is composed by thin parts of the marginals and a few centrals; the exterior appendages of the ambulacral system are present in the form of well developed skeletized digits; on the anterior end of the cup, besides the oral opening, there are one or two apertures more; stem rudimentary.

So far only one family, *Rhipidocystidae* Jkl with a single genus *Rhipidocystis* belongs to this order. This genus, as compared with other Carpoids, e. g. with the Trochocystids and Gyrocystids of the order Cincta, represents a later, though, apparently, less specialized form. The following characters of *Rhipidocystis* may be considered as primitive (unspecialized): strong development of skeletized appendages of the ambulacral system; possible presence of separate anal and oral openings, and of some more apertures on the anterior end of the cup; equal dimensions of the marginal and the central plates of the cup; and almost complete identity of structure of skeleton on the upper and lower surfaces of the cup. The main feature distinguishing *Rhipidocystis* from Cystoids is the absence of thecal pores. It resembles to Cystoids in the presence of strongly developed appendages of the ambulacral system, which in no other Carpoid attain such development. Therefore, *Rhipidocystis* may be as well considered as a form closely allied to Cystoids, not so greatly deviating from common ancestors as the other Carpoids do.

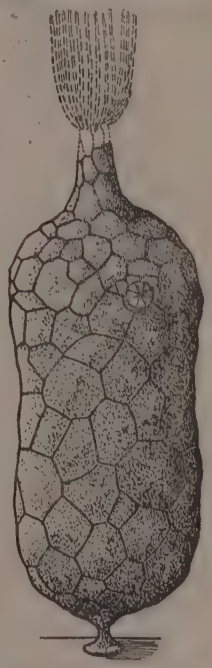

Fig. 2. Reconstruction of *Bockia* n. g.

II. Diagnosis of the new genus *Bockia* (Volb. in coll.), the remains of which were once mistaken by Jaekel for the sac-like roots of *Rhipidocystis*: Cup cucumber-like or sac-like, round in cross-section, sometimes, in large specimens, constricted in the middle, attaining 11.5 cm in height; composed by numerous, irregularly disposed polygonal plates. Surface of the plates covered by tubercles or smooth. Base of the cup formed by a basal circlet usually composed by three plates. The ends of these plates extended downwards, to the facet of the stem. Anus situated on the lateral surface of the upper $^1/_3$ of the cup, and covered by the valvular pyramid. Cup terminates upwards in a proboscis, which must have borne the oral opening, probably some other apertures, and the organs of the ambulacral system. The stem, apparently, was short and terminated in a discoidal root. Ordovician of Leningrad province and Estonia, from Glauconite (B_{II}) up to Itfer (C_3) horizon.

The genus *Bockia* includes 4 species:

1. *Bockia neglecta* n. sp. Cup cucumber-like, in large specimens constricted, height up to 11.5 cm; number of plates up to 100, their diameter up to 15 mm. Plates thin (from $^1/_4$ to $^1/_3$ mm), smooth or covered by sparse, disorderly scattered, not very distinct fine tubercles. Facet for the stem very small, up to 1.5 mm in diameter, pierced by a narrow canal. Anus up to 4 mm in diameter, covered by a pyramid of eight plates. There is a proboscis, skeletized like the cup. A few small low plates evidence the presence of a stem. Abundant in Echinosphaerites limestone (C_1) of Leningrad province, occurs also in Estonia; rarely, in the Glauconite (B_{II}) and Orthoceratites (B_{III}) limestone of Leningrad province.

2. *Bockia sculpta* n. sp. Cup up to 4.2 cm in height, sac-like, without constriction. Number of plates up to 300, most of them are small, the larger ones attain 5 mm in diameter. Ornamentation consists of sparse, very distinct tubercles. Facet for the stem rather large (up to 2 mm). Rare. Echinosphaerites (C_I) and, apparently, Orthoceratites limestone (B_{III}) of Leningrad province and Estonia.

3. *Bockia laevis* n. sp. Cup sac-like up to 4.5 cm or more in height. Plates numerous, of rather uniform dimensions, 5 - 7 and up to 9 mm, comparatively thick (up to $^1/_2$—$^3/_4$ mm) and usually smooth; facet for the stem, up to 4.5 mm in diameter; canal $^3/_4$ mm wide. More common. Kuckers (C_2) and, apparently, Itfer (C_3) horizons of Estonia and Leningrad province.

4. *Bockia cucumis* n. sp. Cup sac-like; plates not numerous, large (up to 10—12 mm); their surface densely and fine granulated and with sparse, distinct tubercles. Rare. Itfer horizon (C_3) of Estonia.

The definition of *Bockia* and its four species shows that this genus has nothing in common with Carpoids, being characterized by a whole number of Cystoid features, such as: 1) the sac-like shape of the cup; 2) the numerous and irregularly disposed cup-plates; 3) the thinness of the cup-plates, the cup being rather large; 4) the presence on the lateral surface of a large anus with a pyramid. However, *Bockia* lacks a very essential character typical for all the Cystoids, viz., the thecal pores, the absence of which is characteristic for the Crinoids.

The combination in *Bockia* of the characters of two classes of Echinoderms may be considered as indicating an intermediate position of this form. It appears best to include *Bockia* into the sub-class Eocrinoidea, erected by Jackel[3] in 1918 and containing primitive Crinoids. Of the forms referred to this group, *Bockia* is most closely allied to the genus *Cryptocrinus* Buch. from the same horizons of the Ordovician of Leningrad province. Following characters relate *Bockia* with *Cryptocrinus*: 1) the absence of thecal pores which in an inconsiderable number were observed by N. Jakovlev [1918, [4]] only in anomalous specimens of *C. laevis* Pand.; 2) the presence of a similar basal circlet; 3) the presence of smooth or finely tuberculated cup-plates with fine growth-lines; 4) the presence on the lateral surface of the upper half of the cup of an anus covered by a pyramid. The main differences between *Bockia* and *Cryptocrinus* consist in that the former developed in the direction of an intensive growth in height, accompanied by the increasing in number of the plates of the cup; the extension of the latter is expressed not only in its shape, but also in the appearence of a proboscis bearing the oral opening and terminating in the appendages of the ambulacral system. While *Cryptocrinus* developed in the direction of diminishing of the number of cup-plates and formation a few (4) circlets, composed by a constant number of plates.

Thus, from the peculiarities of *Bockia* and its resemblance with *Cryptocrinus* we have to consider this genus as a form derivating from true Cystoidea Rhombifera, standing on the boundary between Cystoidea and Carpoidea, as a form which may be provisionally referred, along with *Cryptocrinus*, to the Crinoids—namely to their subclass Eocrinoidea.

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PALEONTOLOGY

A NEW MEMBER OF THE CLASS OPHIOCISTIA SOLLAS (*VOLCHOVIA* n. g.) FROM THE ORDOVICIAN OF LENINGRAD PROVINCE AND CHANGES IN THE DIAGNOSIS OF THIS CLASS

By R. HECKER

(Communicated by A. A. Borissiak, Member of the Academy, 22. III. 1938)

The class Ophiocistia Sollas is for the time being known only by very few remains, from the Silurian and the Middle Devonian, referred to 4 genera—*Eucladia* Woodw., *Sollasina* Fed., *Euthemon* Sollas ([2,6,7]) and *Rhenosquama* Richt. ([5]), distributed between three families: *Eucladiidae* Gregory, *Sollasinidae* Fedotov and *Rhenosquamidae* Richter.

A study of the remains of Echinoderms from the Ordovician of Leningrad province, referred by Jaekel [1900 and 1918 ([3,4])] and Bather [1913([1])] to a new genus *Rhipidocystis* Jkl, showed that the «cup» of this quite erroneously interpreted «compound» form is in reality a test of an Ophiocistid.

The rather abundant though fragmentary material of these Echinoderms comes from more ancient deposits (Glauconite and Orthoceratites limestones of the Ordovician), than the above-mentioned genera, and belongs to two species of a new genus, *Volchovia* n. g. (*V. mobilis* n. sp. and *V. volborthi* n. sp.).

The new genus differs considerably from other genera of Ophiocistids, though it is allied with them more closely than with any other Echinoderm of the sub-type Eleutherozoa to which it undoubtedly refers. The morphological characters of the *Volchovia* are not fully consistent with the current diagnosis of the class Ophiocistia. This is quite natural, since this diagnosis was based on 4 genera only (each including a single species) of an ancient branch of Echinoderms which, I think, is rather rich in various forms.

Thus, the diagnosis of the class Ophiocistia has to be modified.

Diagnosis of the genus *Volchovia*: Test helmet-like, transverse, open below, consisting of an external circlet, composed by 10 pentagonal plates with rounded exterior apexes forming an undulating margin, and a protruding central part formed by plates reaching up to 30 in number. The largest plates of the test attain 3 cm in diameter; they may be either smooth or covered by fine tubercles. On one of the extended lateral surfaces of the test a large anus is situated, covered by the pyramid showing a great number (up to 20) of lobules. Near the anus traces of another aperture are perceived. The lower part of the body, as well as the podia, are not preserved.

Volchovia during its life-time used to creep in search for food over the bottom of the Ordovician sea like the starfishes, with the help of the ambulacral podia (not preserved). The remains of *Volchovia* occur in limestones

abounding with organogenous detritus, and hence we can infer that the animal in question was not a mud eater and possessed a powerful biting and masticating apparatus, which probably resembled the Aristotel's lantern of sea-urchins (compare with *Eucladia*, *Sollasina* and *Euthemon*).

Distinctive characters of the two species of *Volchovia*:

1. *Volchovia mobilis* n. sp. Diameter of the test never exceeds 6—7 cm. Dimensions of plates may be up to 1.5—2 cm, they are comparatively thin, their surface being smooth, or—very rarely—covered by fine tubercles. Glauconite limestone (zones $B_{II\beta}$ and $B_{II\gamma}$); rarely Orthoceratites limestone (B_{III}).

2. *Volchovia volborthi* n. sp. Diameter of the test up to 8—9 cm. The dimensions of plates attain 3 cm; they are somewhat thicker than the plates

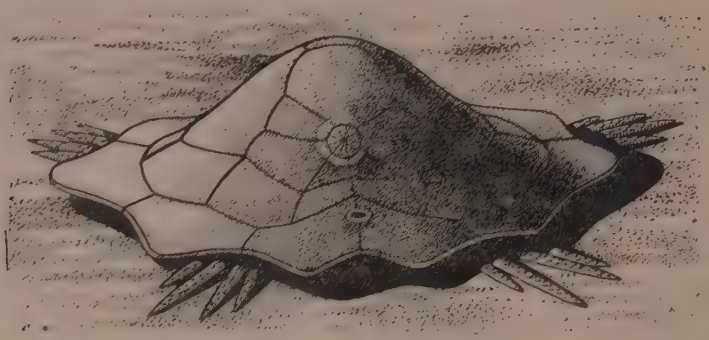

Fig. 1. Reconstruction of *Volchovia* n. g.

of *V. mobilis* and covered with densely set fine tubercles. Orthoceratites limestone (B_{III}).

Volchovia in following characters is allied to Ophiocistia:

1. A discoidal form of the body with skeleton, though the latter does not cover the lower surface of the body.

2. Characters inferred from the structure of the skeleton:

a) the mobile mode of life and connected with this

b) the surmised presence of ambulacral podia of rather considerable dimensions, with the help of which the animal could move, and

c) the supposed presence of a maxillary apparatus.

However, the genus under description displays following characters, not occurring in other forms of Ophiocistia and not included into the diagnosis of this class:

1. In *Volchovia* only the upper and lateral parts, but not the whole of the body, are covered by a continuous skeleton composed by polygonal plates.

2. The ambulacral podia were very weakly, if at all skeletized.

3. A very large anus covered by a massive pyramid is present, which character was never observed in other members of Ophiocistia.

Therefore, referring *Volchovia* to the class Ophiocistia, it is necessary to modify the diagnosis of the latter, formulating it—until new forms of Ophiocistia will be discovered—as follows:

Class Ophiocistia Sollas, 1899, 1912, em. Fedotov, 1926. Extinct Asterozoa with discoidal body, either entirely covered by polygonal plates or with but slightly skeletized, actinal (ventral) surface. No arms, but in each radius from the lower surface of the body (the disc) extend several pairs of very large ambulacral podia, usually covered by external skeleton and

serving as walking feet. Mouth lying in peristome with 5 movable powerful jaws. Anus not always distinctly defined; probably in some cases absent; when present, it is covered by a valvular pyramid. Madreporite large, actinal, in most forms not observed. Ordovician—Middle Devonian.

The differences between *Volchovia*, on the one side, and *Eucladia*, *Sollasina* and *Euthemon*, on the other, are more essential than the differences between the latter three genera. Hence, *Volchovia* must be considered as a form evidently referring to a new family, to which the name *Volchoviidae* is appropriated. The diagnosis of this family is identical with the diagnosis of the genus *Volchovia*.

As archaic features of the *Volchovia*, which is the earliest of all the known Ophiocistids, may be tentatively mentioned the following characters:

1. The presence of a large anus covered by a pyramid like the anal pyramids of Cystoidea, while in the Gotlandian Ophiocistids there is apparently no anus altogether.
2. No skeleton on the lower part of the body.
3. Not skeletized, or but slightly skeletized ambulacral podia.

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PALEONTOLOGY

CONTRIBUTIONS TO THE QUESTION CONCERNING THE EVOLUTION OF CARBONIFEROUS (BASHKIRIAN) *CHORISTITES*

By S. SEMICHATOVA

(Communicated by A. A. Borissiak, Member of the Academy, 24. III. 1938)

The Bashkirian beds have been established in the Carboniferous section on the basis of the peculiarities of their Brachiopods. This fauna is being studied by me at present. The first fascicle of its monographical description has been presented shortly for print to the Paleontological Institute of the Academy of Sciences of the USSR*. This work deals with Bashkirian *Choristites*. It contains the description of 22 species and varieties, and 5 forms described as aff. of definite species or without any specific determination. The fauna under examination is derived from different localities on the western slope of the Ural Mountains and from the Southern Ferghana.

The List of Species Described. Group of *Choristites bisulcatiformis* Semich.: 1) *Ch. bisulcatiformis* Semich.; 2) *Ch. bisulcatiformis* Semich. var. *bifurcata* var. n.; 3) *Ch. bisulcatiformis* Semich. var. *rotundata* var. n.; 4) *Ch. bisulcatiformis* Semich. var. *anomala* var. n. 5) *Ch.* aff. *bisulcatiformis* Semich.; 6) *Ch. fasciculatus* sp. n.; 7) *Ch. praepavlovi* Semich.; 8) *Ch. robustus* sp. n.; 9) *Ch. urta-ayryensis* sp. n.; 10) *Ch. yanghukouensis* Chao em. n. var. *crassicostata* var. n.; 11) *Ch. yanghukouensis* Chao em. n. var. *bashkirica* var. n.; 12) *Ch. transversus* sp. n.; 13) *Ch. yakhjaensis* sp. n.; 14) *Ch. regularus* sp. n.; 15) *Choristites* sp. A; 16) *Choristites* sp. B.

Group of *Ch. elegans* sp. n.: 17) *Ch. elegans* sp. n.; 18) *Choristites* aff. *elegans* sp. n.

Group of *Choristites uralicus* Leb.: 19) *Ch. lebedevi* Frcks.; 20) *Ch. lebedevi* Frcks. var. *laticostata* var. n.; 21) *Ch. uralicus* Leb. var. *asiatica* Frcks.

Group of *Choristites obrotundus* sp. n.: 22) *Ch. obrotundus* sp. n.; 23) *Ch. obrotundus* var. *angulatisinuata*, sp. et var. n.; 24) *Choristites* sp. C.

Group of *Choristites priscus* Eichw.; 25) *Ch. mansuyi* Chao; 26) *Ch. mansuyi* Chao var. *transversa* var. n.; 27) *Choristites andygensis* sp. n.

The Bashkirian fauna has not been described till now except a short description of three species of Spiriferidae, published by me in 1936 (2). The Brachiopods from the beds, underlying the Bashkirian beds, have been described, with respect to the western slope of the Ural Mountains, only from one locality in the Northern Ural (4). The Brachiopods (and the Foraminifera)

* Brachiopods from the Bashkirian beds, fasc. 1, genus *Choristites* Fischer. Delivered at the meeting of the Paleontological Institute on March 3, 1938.

Fig. 1.

1—Sp. pseudotrigonalis var. *furcata* m. Thin section 1421a. *2—Sp. praebisulcatus* m. Thin section 1178a (a little schematic). *3—Ch. bisulcatiformis* Semich. var. *anomala* var. n. Thin section 47a *4—Ch. urta-ayryensis* sp. n. Thin section 67a (a little schematic). *5—Choristites* sp. B ex gr. *Ch. bisulcatiformis* Semich. Thin section 4a. *6—Ch. bisulcatiformis* Semich. Thin section 7a. *7—Ch. bisulcatiformis* Semich. var. *bifurcata* var. n. Thin section 250a. *8—Ch. yanghukouensis* Chao em. n. var. *bashkirica* var. n. Thin section (23) 17a. *9—Ch. obrotundus* sp. n. Thin section (5) 131a. *10—Ch. priscus* Eichw. var. *uralica* var. n. Thin section 769a.

of the Moscovian beds overlying the Bashkirian in the Urals are known till now only in lists (3). The Bashkirian Brachiopods obtained from regions neighbouring geosynclines exhibit no species in common with the Moscovian fauna, well known in the Russian platform. These facts account for the specific isolation of the Bashkirian *Choristites*, well displayed in the list above by the considerable per cent of new forms.

The stratigraphic considerations based on the study of Bashkirian *Choristites* will be presented in another paper. The study of Bashkirian *Choristites* gives the possibility to elucidate some interesting data concerning the history of development of the genus *Choristites* Fischer and of the genetic relations of some *Choristites* to the Lower Carboniferous *Spirifer* s. str. These data are summarized in the table.

In this table only those forms are placed the position of which in the Bashkirian section can be firmly established. The transverse sections of the apical part of ventral valve of Spiriferidae given in this table represent thin slides of corresponding forms drawn from photographs.

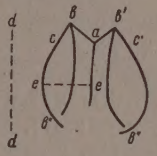

Fig. 2.

According to this table we are able to trace a line of development between the most ancient representatives of *Choristites* of the *Ch. bisulcatiformis* group from the base of Bashkirian beds and the members of the same group from the upper portion of the same series of beds, as well as the representatives of the *Ch. obrotundus* group, met with in the upper half of this series, and representatives of the *Ch. priscus* group, appearing in the uppermost horizons of the Bashkirian series and attaining their chief development in beds of Moscovian age. The absence of corresponding data concerning the Spiriferidae from the beds underlying the Bashkirian in the Urals compelled me to use for comparison data, concerning the Lower Carboniferous Spirifers from the Moscow basin (Serpuchov beds).

This part of the evolutionary sequence has to be verified by subsequent studies founded on material derived from a series of consecutive beds overlying one another, when the facial character of the fauna would be approximately the same in all the portions of the section and the relations of separate faunules as to their respective age would be fully clear.

Among the Spirifers of Lower Carboniferous beds I have established the subgroup of *Sp. pseudotrigonalis* m., the members of which differ somewhat in the characters of their inner structure from other representatives of Lower Carboniferous Spirifers (Table, Fig. *1* and *2*). As regards the outer features the members of this subgroup are characterized by strong rarely bifurcating ribs which are reflected on the inner sides of their valves. The bifurcation of the ribs usually does not affect in these Spirifers more than the anterior half of their shell. The *Choristites* met with in the lowermost portion of Bashkirian beds stand very near as to their inner structure to the representatives of *Sp. pseudotrigonalis* subgroup, though distinguished from the latter by features approaching the inner characteristics of typical Moscovian *Choristites* (Table, Fig. *3* and *4*). The bases of their apical plates are relatively shorter, and not so markedly turned aside from each other; the ratio of width of apical plates to their height is different.

Fig. 2 serves to the elucidation of terms used in this paper; *abc* and *ab'c'*—bases of apical plates; *dd*—their height; *ee*—their width; *bb'* and *b'b''*—median lines. The terms are used exactly in the same sense in all my previous papers. With respect to the character of ribs and of their bifurcation the forms under consideration differ but slightly from some of Lower Carboniferous Spirifers, with which they also exhibit affinities as to the inner structure (*Sp. pseudotrigonalis* var. *furcata*, *Sp. praebisulcatus*).

The data at hand give the possibility to point out a definite branch among the Spiriferidae, which unites some representatives of *Spirifer* s. str. and *Choristites* Fischer; this branch might be subdivided into two genetic lines (Table, Figs *2*, *4*, *5*, *7*, *8*, *9*, *10*—one line, Figs *1*, *3*, *4*—another line). On the basis of new data the evolution of this branch may be traced during the end of Lower Carboniferous and throughout the Middle Carboniferous time. We have no data as to the character of its development during the deposition of beds with *Sp. bisulcatus*, absent in Moscow basin and insufficiently studied as to their fauna in other localities. The lack of full consecutiveness in the variations between Spirifers of *Sp. pseudotrigonalis* subgroup and *Choristites* of *Ch. bisulcatiformis* group agrees well with the supposition advanced a few years ago [1] as to a break existing between Serpuchovian and Bashkirian time—break corresponding to the deposition of beds with *Sp. bisulcatus* in Donetz basin and in some other localities.

Besides the branch of Spiriferids the development of which has just been shortly discussed, there might be suggested among Bashkirian *Choristites* the presence of another branch, characterized by high and thin apical plates and by a more strongly pronounced plication of the inner surface of valves. To this series of forms belong *Ch. elegans*, *Ch.* aff. *elegans* and partly *Ch. praepavlovi*. But this last form exhibits in other respects connection with the *Ch. bisulcatiformis* group and is not at present separated from it. More ample material would possibly enable us to speak more confidently of the existence of this second branch and to trace its connection with those lower Carboniferous Spirifers which have high and thin apical plates (*Sp. gamma* m. and some others).

The work concerned with the question of the establishment of genetic lines among the Spiriferidae of Lower and Middle Carboniferous is only at its beginning at the present time. The rich material collected during these last years owing to the large scale, on which geological investigations are being carried on in our Union, gives a good basis for this work. It is to be hoped that further study will enable us to verify and to develop the results of my present work and will bring us nearer to the solution of the problem of establishing natural generic groups among Brachiopods.

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